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Surgery with Coefficients

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INTRODUCTION

This set of notes is derived from a seminar given at the University of Michigan in 1973, and portions of the author's doctoral thesis. It is intended to give a reasonably complete and self-contained account of surgery theory modulo a set of primes.

The first three chapters contain the background material necessary to describe the theory. Chapter 1 is mainly definitions and notation and contains no new ideas, with the exception of relative localization and colocalization of spaces. Included is a sketch of the immersion classification theorem of Hirsch and Haefliger-Poenaru.

Chapter 2 contains the theory of local Whitehead torsion. The definition differs from the one given by Cappell and Shaneson, but is justified by a Whitehead-type local collapse-expansion theorem. Chapter 3 discusses the theory of spaces which satisfy Poincare duality with coefficients in a ring, including the construction of a local Spivak normal fibration. Normal invariants modulo a set of primes are described and the homotopy groups of the classifying space G_p/H are computed.

Chapter 4 contains the main surgery obstruction theorem. Briefly, groups are constructed to measure the obstruction to finding a homotopy equivalence (over a ring and

with given torsion) cobordant to a given map. Below the middle dimension, the technique is due to Milnor and Wallace. Considering homotopy equivalences over the integers, the simply connected case is essentially done by Kervaire and Milnor, and globalized by Browder and Novikov; the general case is due to Wall. We show that the obstruction lies in a Wall group of a localized group ring.

Surgery over a field was first considered by Petrie and Passman, and Miscenko noticed that Wall's groups behaved nicely away from the prime 2. More recently, Connelly, Giest and Pardon have considered rational surgery (in the non-simple case), and the methods of Cappell and Shaneson (which uses rings with a local epimorphism $\mathbb{Z}\pi \to R$) also apply. The general case, with rings of the form $R\pi$, is due to the author in his thesis.

Chapter 5 gives the geometric definition of surgery groups, and the generalization to manifold n-ads. Quinn's approach is also briefly discussed. Finally, the periodicity theorem, in the non-simple case, is proved.

Chapter 6 describes the result of changing rings in surgery groups by means of a long exact sequence. Corollaries include a Rothenberg-type sequence, the general periodicity isomorphism and determination of the kernel of $L^s_{2k-1}(\mathbb{Z}\pi) \to L^s_{2k-1}(\mathbb{Q}\pi)$, π finite, by simple linking forms, generalizing the original odd-dimensional surgery obstructions due to Wall and clarified by Connelly.

Finally, five appendicies are included: Whitehead torsion notions for n-ads, the algebraic construction of $L_n(\pi\to\pi';R)$, the computation of $L_n(\mathbb{Z}^k;R)$, surgery on embedded manifolds, and homotopy and homology spheres. The reference has been arranged into categories. Undoubtedly, some errors and omissions have occurred in this arrangement, but I hope the general drift is helpful to the reader.

A number of people have been of great help in writing these notes. I am indebted to my thesis advisor C.N. Lee for many helpful suggestions and discussions. I would also like to thank Dennis Barden, Allan Edmonds, Steve Ferry, and Steve Wilson, who participated in the seminar, Frank Raymond, Jack Mac Laughlin and W. Holsztynski.

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