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Unipotent Algebraic Groups

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P R E F A C E

The geometry and group theory of unipotent algebraic groups over an arbitrary ground field were successfully pioneered by Rosenlicht in the late fifties and early sixties. In the subsequent years not very much was added to the knowledge in this area, with only a few notable contributions such as those by Russell and by Tits. Lately, however, there have been indications of growing interest in this and related subject areas (affine space, its automorphisms, purely inseparable cohomology theories, ...). Even as the present paper was undergoing the final redaction, a graduate student in Tokyo settled our conjecture in Section 5 by constructing an elaborate counter-example; one of the coauthors established the absence of nontrivial separable forms of the affine plane, confirming an earlier announcement of Shafarevich; another found a description of the category of all commutative affine group schemes over an imperfect field by extending Schoeller's work; and still another obtained an algebraic characterization of the affine plane.

The material presented here might be made into two or three separate research papers of a more polished character. Instead, in view of the rapid developments as indicated above and because of our belief in the unity behind our work, we have chosen to publish our results as one whole and as quickly as possible. We are thankful to the editors and the publishers of the Lecture Notes series for providing us with an ideal outlet for our joint work. It is our sincere hope that this publication will serve to stimulate further research in this field full of deep and fascinating problems.

Finally, our grateful acknowledgements are due to the Research Institute for Mathematical Sciences, Kyoto University for the hospitality extended to one of us while the research for the present paper was conducted during the year 1972-73; to the young ladies on the Institute's staff for the carefull and efficient typing of the manuscript; and to the National Science Foundation for partially supporting the final preparation of the manuscript through a research grant.

June 1974

The Coauthors

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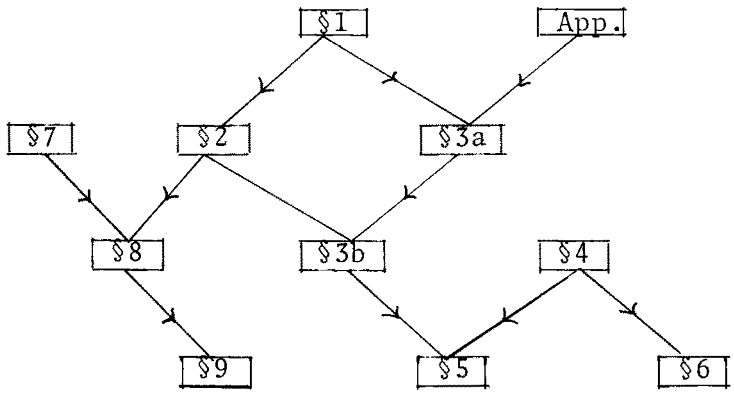

§3a = 3.1 to 3.4; §3b = 3.5 to 3.7