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15

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Commutative group schemes

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Introduction

We restrict ourselves to two aspects of the field of group schemes, in which the results are fairly complete: commutative algebraic group schemes over an algebraically closed field (of characteristic different from zero), and a duality theory concerning abelian schemes over a locally noetherian prescheme. The preliminaries for these considerations are brought together in chapter I.

SERRE described properties of the category of commutative quasi-algebraic groups by introducing pro-algebraic groups. In characteristic zero the situation is clear. In characteristic different from zero information on finite group schemes is needed in order to handle group schemes; this information can be found in work of GABRIEL. In the second chapter these ideas of SERRE and GABRIEL are put together. Also extension groups of elementary group schemes are determined.

A suggestion in a paper by MANIN gave crystallization to a feeling of symmetry concerning subgroups of abelian varieties. In the third chapter we prove that the dual of an abelian scheme and the linear dual of a finite subgroup scheme are related in a very natural way. Afterwards we became aware that a special case of this theorem was already known by CARTIER and BARSOTTI. Applications of this duality theorem are: the classical duality theorem ("duality hypothesis", proved by CARTIER and by NISHI); calculation of $\mathrm{Ext}(\underline{\underline{G}}_a, A)$, where A is an abelian variety (result conjectured by SERRE); a proof of the symmetry condition (due to MANIN) concerning the isogeny type of a formal group attached to an abelian variety.

As we said before, our results originate from work of SERRE and GABRIEL. Besides that of course the ideas and results of GROTHENDIECK were indispensable. I am greatly indebted to J.-P.Serre, from whom I received valuable suggestions and helpful correspondence, to P.Gabriel, who, having read the manuscript, proposed many improvements and who gave precious information, and to J.P.Murre for his continuous interest in my work.

Terminology and notations. We say that a diagram is exact, if all columns and all rows are exact sequences. The terms injective, surjective and bijective are to be taken in the set-theoretical sense, while monomorphic, epimorphic and isomorphic are to be taken in the categorical sense (an epimorphic ring-homomorphism needs not to be surjective, a surjective morphism of schemes needs not to be an epimorphism). Working with preschemes, we use Mor(- , -) in order to indicate a set of morphisms (this notation differs from the corresponding one in EGA), while working with group schemes, we use Hom(- , -) in order to denote a set of homomorphisms (in this case HOM(- , -) seems to be the current notation). If E and F are modules over a ring A, we write $\mathrm{Hom}_A(E,F)$ for the set of A-homomorphisms from E into F. If B and C are A-algebras we write $\mathrm{RHom}_A(B,C)$ for the set of A-algebra homomorphisms from B into C. The multiplicative group of units of a ring A with identity element we denote by A^*. We use $X \in (\mathrm{Sch}/S)$, or $X \in (\mathrm{Sch}/A)$ in case $S = \mathrm{Spec}(A)$, as an abbreviation for "X is a prescheme over S"; we consider locally noetherian preschemes only, if not mentioned otherwise.

Square brackets and abbreviations (such as GP and EGA) will be used to indicate bibliographical references.

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Contents

Chapter I Preliminaries

Chapter II Algebraic group schemes

Chapter III Duality theorems for abelian schemes