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Theory of Hopf Algebras
Attached to Group Schemes



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PREFACE

The purpose of these notes is to develop systematically a theory of Hopf algebras attached to group schemes over an algebraically closed field. It is well known that Lie algebras attached to algebraic groups play very important roles in the theory of algebraic groups over a field of characteristic zero, but on the contrary they are powerless in positive characteristic cases. This is essentially due to the fact that the enveloping algebra of the Lie algebra attached to an algebraic group over a field of positive characteristic is a proper subalgebra of the Hopf algebra attached to it in contrast to the case of characteristic zero, where the Hopf algebra attached to an algebraic group coincides with the enveloping algebra of the Lie algebra attached to it. In other words Lie algebras attached to algebraic groups over a field of positive characteristic p do not give any information on higher order infinitesimals than p . Therefore it would be natural to study rather Hopf algebras than Lie algebras, if we want to develop an infinitesimal theory of group schemes over a field of arbitrary characteristic.

In the first and second sections we give basic definitions and results on group schemes over an algebraically closed field which are necessary in the later sections. In particular Theorem 2.12, given originally in [31], plays an essential role in the whole theory. The next three sections are devoted to the theory of

coalgebras, bigebras and Hopf algebras over a field coming from [10], [21], [24], [27] and [32]. However the proof of the structure theorems 5.3 and 5.4 of cocommutative and colocal bigebras will be given in Appendix I, because it is rather complicated and long. The materials of §6 and §7 come from [3], [4], [6], [8], [29] and [30], where some basic properties of formal groups are shown in connection with Hopf algebras and group schemes. The next seven sections consist of the contents of the paper [32] and are devoted to the theory of algebraic Hopf subalgebras of Hopf algebras attached to group schemes which corresponds to the theory of Lie algebras attached to algebraic groups over a field of characteristic zero developed in [2], [7] and [11]. It should be noted that the articles [9], [10] and [25] contain similar results to ones in these sections for some special cases. In §15 we shall show how to get most results on algebraic Lie subalgebras of Lie algebras attached to algebraic groups over a field of characteristic zero from our results on algebraic Hopf subalgebras given in the preceding sections. In §16 we shall give the theory of left invariant semi-derivations on group varieties over a field of positive characteristic which was developed originally in [1] and then in [28]. The next two sections are the main contents of the paper [5] where interesting results on quotient varieties of group varieties by group subvarieties are given. In the last section we shall study purely inseparable isogenies of group varieties in terms of Hopf algebras following after [28] and [15]. Appendix II is devoted to the proof of Proposition 14.10 coming from [24], and we shall

give a proof of Proposition 19.11 in Appendix III by the idea in [22] and [23].

There are many interesting results on Hopf algebras attached to group schemes not contained in these notes, for which the articles [1], [9], [10], [13], [29] and [30] may be referred. Making these notes, the author tried to let these self-contained as possible as he could, but the reader would be assumed to have some basic knowledge on algebraic geometry and commutative algebra, for instance, in [12], [16], [17], [18] and [33].

These notes are based on the lectures given by the author at Genova in the first half of 1976. Here the author would like to express his hearty thanks to the attendances of the lectures. In particular he appreciates deeply that P. Salmon and D. Arezzo, who attended constantly to the lectures, helped him to stay comfortably at Genova, because it would be impossible to complete this work without their help. Lastly the author expresses his thanks to Y. Ishibashi and A. Ooishi for reading the manuscript and pointing errors, and also to C. Hayashi for her excellent typewriting.

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