Lecture Notes in Physics

Springer
Berlin
Heidelberg
New York
Barcelona
Hong Kong
London
Milan
Paris
Singapore
Tokyo

Physics and Astronomy ONLINE LIBRARY

http://www.springer.de/phys/

The Editorial Policy for Monographs

The series Lecture Notes in Physics reports new developments in physical research and teaching - quickly, informally, and at a high level. The type of material considered for publication includes monographs presenting original research or new angles in a classical field. The timeliness of a manuscript is more important than its form, which may be preliminary or tentative. Manuscripts should be reasonably self-contained. They will often present not only results of the author(s) but also related work by other people and will provide sufficient motivation, examples, and applications.

Acceptance

The manuscripts or a detailed description thereof should be submitted either to one of the series editors or to the managing editor. The proposal is then carefully refereed. A final decision concerning publication can often only be made on the basis of the complete manuscript, but otherwise the editors will try to make a preliminary decision as definite as they can on the basis of the available information.

Contractual Aspects

Authors receive jointly 30 complimentary copies of their book. No royalty is paid on Lecture Notes in Physics volumes. But authors are entitled to purchase directly from Springer-Verlag other books from Springer-Verlag (excluding Hager and Landolt-Börnstein) at a $33\frac{1}{3}$% discount off the list price. Resale of such copies or of free copies is not permitted. Commitment to publish is made by a letter of interest rather than by signing a formal contract. Springer-Verlag secures the copyright for each volume.

Manuscript Submission

Manuscripts should be no less than 100 and preferably no more than 400 pages in length. Final manuscripts should be in English. They should include a table of contents and an informative introduction accessible also to readers not particularly familiar with the topic treated. Authors are free to use the material in other publications. However, if extensive use is made elsewhere, the publisher should be informed. As a special service, we offer free of charge LaTeX macro packages to format the text according to Springer-Verlag's quality requirements. We strongly recommend authors to make use of this offer, as the result will be a book of considerably improved technical quality. The books are hardbound, and quality paper appropriate to the needs of the author(s) is used. Publication time is about ten weeks. More than twenty years of experience guarantee authors the best possible service.

LNP Homepage (http://www.springerlink.com/series/lnp/)

On the LNP homepage you will find:
—The LNP online archive. It contains the full texts (PDF) of all volumes published since 2000. Abstracts, table of contents and prefaces are accessible free of charge to everyone. Information about the availability of printed volumes can be obtained.
—The subscription information. The online archive is free of charge to all subscribers of the printed volumes.
—The editorial contacts, with respect to both scientific and technical matters.
—The author's / editor's instructions.

Mirko Degli Esposti   Sandro Graffi (Eds.)

The Mathematical Aspects of Quantum Maps

Springer

Editors

Mirko Degli Esposti
Sandro Graffi
Università di Bologna
Dipartimento di Matematica
Piazza di Porta San Donato 5
40127 Bologna, Italy

Cataloging-in-Publication Data applied for

A catalog record for this book is available from the Library of Congress.

Bibliographic information published by Die Deutsche Bibliothek

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at http://dnb.ddb.de

ISSN 0075-8450
ISBN 3-540-02623-1 Springer-Verlag Berlin Heidelberg New York

Springer-Verlag Berlin Heidelberg New York
a member of BertelsmannSpringer Science+Business Media GmbH

http://www.springer.de

Printed in Germany

Typesetting by the authors/editors
Camera-data conversion by Steingraeber Satztechnik GmbH Heidelberg
Cover design: *design & production*, Heidelberg

Printed on acid-free paper
55/3141/du - 5 4 3 2 1 0

Foreword

Quantum chaos usually means the investigation of quantum mechanical properties of classically chaotic systems. Expressed differently: what are the manifestations, if any, of chaotic behavior in classical systems on their quantum counterparts?

This question has obviously been thoroughly investigated since the early days of quantum mechanics, for instance, in the classical papers of Von Neumann on quantum ergodicity [14] and of Wigner on the quantum analog of the Boltzmann equation [15]. More recent approaches to this general question can be found for instance in [8], [9] and [10].

This kind of issue has obvious foundational aspects. For example, the need to justify quantum statistical mechanics in the same way as classical chaos is the basis for justifying classical statistical mechanics. However, investigations in quantum chaos also arise from questions of direct and important physical relevance: the spectral statistics in complex nuclei and the "stochastic" ionization of the hydrogen atom in a microwave field, to mention only two. The study of spectral statistics led to the formulation of standard conjectures on the probability distribution of the eigenvalue spacing. The Poisson probability distribution should be valid for the quantized counterpart of the classically integrable systems (Berry-Tabor conjecture, formulated in the seventies [4], see also [11,12]) and the COE/CUE (Circular Orthogonal/Unitary Ensemble) probability distribution [30] for the quantized counterpart of the classically chaotic systems (Bohigas-Giannoni-Schmit conjecture, formulated in the eighties [1]). The numerical evidence for the validity of this conjecture is so convincing that the spacing distribution among quantum levels is considered as a characteristic property of quantum chaotic systems even though no example is known where a proof can be obtained (see for example [8,10] and references therein). Usually the spacing distribution is explored through some spectral indicators such as the form factor (namely the Fourier transform of the two-point correlation function), according to the COE/CUE statistics (yielding the so-called level repulsion)

On the other hand, the study of time-dependent models (such as the kicked rotator) or other simplified versions of the hydrogen atom in a microwave field produced a second important numerical discovery, the quantum suppression of classical chaos. The numerical evidence (mainly on the kicked rotator and its variants) shows that classical diffusive behavior at large times is transformed into localized behavior after quantization ([6,7], for a general review see [5] and references therein).

The mathematical understanding of these and many other, "quantum chaotic" features is not easy for several conceptual and technical reasons. First, it has to be recalled that chaotic behavior in classical systems is described in terms of well-defined mathematical notions derived from ergodic theory (ergodicity, mixing, K-property, exponential decay of correlations, etc.), which ultimately depend on the possibility of localizing the initial conditions with arbitrary accuracy. Typically, the sensitive dependence on initial conditions is the standard dynamical mechanism generating chaotic behavior. The indeterminacy principle altogether excludes this mechanism in quantum mechanics and, moreover, makes the classical limit a much subtle problem in quantized "chaotic" systems than in quantized integrable ones. Another way of looking at the nontriviality of the classical limit in this situation is the noncommutativity of the two limits $t \to \infty$ and $\hbar \to 0$ because the very definition of chaotic behavior in classical systems actually requires the limit $t \to \infty$.

The iteration of area preserving maps, which also sometimes describe Hamiltonian systems at discrete times, represent the basic examples of ergodic theory (the Arnold cat, the baker's transformation and the sawtooth map, just to mention few) and therefore the most thoroughly studied examples of non-trivial discrete-time evolution which may give rise to chaotic behavior. Since the phase space of classical discrete dynamics is compact, again by the indeterminacy principle its quantum counterpart will allow only a finite number of states (roughly speaking, a quantum state occupies a volume of size at least $\hbar$) and therefore the quantum evolution takes place within a finite dimensional Hilbert space. The classical limit corresponds to the Hilbert space dimension tending to infinity. The investigation of the quantum maps requires, therefore, the preliminary clarification of several nontrivial mathematical questions, beginning from an unambiguous implementation of a quantization procedure. It will be described how the quantization defines the quantum evolution as the iteration of a unitary $N \times N$ matrix (the propagator), N being proportional to the inverse Planck constant. The quantized maps thus represent the simplest testing ground for the verification not only of the above mentioned conjecture on spectral statistics, but also for novel concepts isolated in the context of the classical limit of quantized chaotic systems, such as (unique) quantum ergodicity, and "scarring" of the eigenfunctions. Moreover, quantum maps is one of the most relevant physical examples where techniques from analytic and algebraic number theory emerge naturally.

Finally, it is also important to notice that quantum maps, seen as unitary transformations on finite Hilbert spaces (of dimension $N = 2^n$) could be interpreted as the implementation of particular quantum gate acting on the space of qubits, as defined in the theory of quantum computation. The exploration of the mathematical properties of quantum maps could shed some light on the understanding of the implementation of new and more efficient quantum algorithms.

The present volume deals precisely with the mathematical aspects of quantum maps mentioned above. Its purpose is to give a broad and in-depth overview of the mathematical problems arising in quantum maps, with a particular emphasis on their physical origin and on their numerical investigation, from the beginning (which may be traced back to the classical 1980 paper of Berry and Hannay on the quantization of linear hyperbolic symplectic maps over the torus [3] and to the 1983 paper by Balazs and Voros on the quantization of the baker map [4]) to the present state of the art. The contents of the present volume reflects the approach chosen to achieve that purpose. Three main topics are covered in full detail. The first topic presented , by A. Knauf, is an overview of that part of classical dynamical systems most relevant to quantum chaos in general and quantum maps in particular, mainly ergodic theory. The second topic, by Z.Rudnick, is an introductory presentation of some basic concepts and techniques out of number theory.

The third topic, by the editors of this book, concerns the general quantization procedure, including observables, for the symplectic maps, both linear (e.g. the Arnold cat) as well as piecewise linear (e.g. the baker's transformation) and the related mathematical results concerning the recovery of the classical chaotic behavior at the classical limit. A fourth contribution, by A. Bäcker, is an introduction to the the numerical aspects in quantum chaos, in particular eigenvalue and eigenfunction computations for chaotic quantum systems, such as discrete maps or two-dimensional billiards. The classical dynamics of two-dimensional area-preserving maps on the torus is illustrated using the standard map and a perturbed cat map. The quantization of area-preserving maps given by their generating function is discussed and for the computation of the eigenvalues a computer program in Python is presented. In particular, the author illustrates the eigenvalue distribution for two types of perturbed cat maps, one leading to COE and the other to CUE statistics. For the eigenfunctions of quantum maps, the author studies the distribution of the eigenvectors and compare them with the corresponding random matrix distributions.

The final contribution, by R. Artuso, is meant to illustrate some features of deterministic transport in chaotic systems. The first part of this contribution deals with the case of *hyperbolic* systems, where typically normal diffusion is observed, while the second part explores *weakly* chaotic systems, where long trappings near regular phase-space regions may induce anomalies in diffusive properties. Examples of analytic calculations are given in the framework of *cycle expansions*, a general technique for obtaining chaotic averages.

References

1. O. Bohigas, M.-J. Giannoni and C. Schmit, *Characterization of chaotic quantum spectra and universality of level fluctuation laws* Phys. Rev. Lett. **52** , 1–4 (1984).
2. M.V. Berry and J.H. Hannay, *Quantization of linear maps on a torus – Fresnel diffraction by a periodic grating*, Physica D **1** (1980), 267–291.
3. N.L. Balazs, A. Voros, *The quantized Baker's transformation*, Annals of Physics **190** (1989), 1–31.
4. M.V. Berry and M. Tabor *Level clustering in the regular spectrum* Proc. Roy. Soc. A **356**, 375-394 (1977).
5. G. Casati and J. Ford *Stochastic behavior in classical and quantum Hamiltonian systems*, Proc. Como Conf. 1977. Lect. Notes Physics **93**, Springer-Verlag Berlin Heidelberg New York (1977).
6. B.V. Chirikov and G.M. Zaslavskii, Sov. Phys. Uspekhi **14**, 549 (1972).
7. B.V. Chirikov and G.M. Zaslavskii, Sov. Phys. JETP **46**, 1094 (1977).
8. M.C. Gutzwiller *Chaos in classical and quantumMechanics*. Springer-Verlag Berlin Heidelberg New York, IAM 1, (1990).
9. M.-J. Giannoni, A. Voros and J. Zinn-Justin *Chaos and Quantum Physics: Les Houches 1989*, North-Holland (1991).
10. F. Haake *Quantum signatures of chaos*, Springer-Verlag Berlin Heidelberg New York (2001).
11. J. Marklof *Level spacing statistics and integrable dynamics*, Proceedings of the XIIIth International Congress on Mathematical Physics, London 2000, 359–363 (2001).
12. J. Marklof *The Berry-Tabor conjecture*, Proceedings of the 3rd European Congress of Mathematics, Barcelona 2000, Progress in Mathematics **202**, 421–427 (2001).
13. M.L. Mehta *Random matrices*. Academic Press,New York and London, (1991).
14. J.von Neumann *Beweis des Ergodensatzes und des H-Theorems in der Neuen Mechanik* Zschr.f.Physik **57**, 30–70 (1929).
15. E.P. Wigner, Phys. Rev. **40**, 749 (1932).

Preface

The present volume represents the collected lectures of a Summer School on mathematical aspects of quantum maps held at Universitá di Bologna (Villa Gandolfi-Pallavicini) from September 1 to September 11, 2001.

The Summer School has been organized as an institutional activity of the EC-supported European Research and Training Network "The Mathematics of Quantum Chaos", whose nodes are at the Universities of Bristol, UK (coordinating node), Bologna (Italy), Paris XI (France), Tel Aviv (Israel), Ulm (Germany) and Uppsala (Sweden). Its purpose was to give a broad and in-depth overview of the mathematical problems arising in quantum maps, with a particular emphasis on their physical origin and numerical investigation.

Acknowledgement

This work has been supported by the European Commission under the Research Training Network (Mathematical Aspects of Quantum Chaos) n$\underline{\text{o}}$ HPRN-CT-2000-00103 of the IHP Programme.

Bologna, Italy
January 2003

Mirko Degli Esposti
Sandro Graffi

Contents

Introduction to Dynamical Systems
Andreas Knauf 1

Number Theoretic Background
Zeév Rudnick 25

Mathematical Aspects of Quantum Maps
Mirko Degli Esposti, Sandro Graffi 49

Numerical Aspects of Eigenvalue and Eigenfunction Computations for Chaotic Quantum Systems
Arnd Bäcker 91

From Normal to Anomalous Deterministic Diffusion
Roberto Artuso 145