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Tauberian
Remainder Theorems



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PREFACE

The main part of these notes consists of the material in a series of lectures I gave at the Institute of mathematical sciences "Matscience" in Madras in January 1969 and which has been available in a mimeographed report from that institute. In this edition I have added some topics I have given in lectures at other places. Many theorems represent unpublished work of mine and I have also included some references to work of my students.

The main theme is the application of Wiener's general method to different kinds of tauberian problems, in particular to remainder problems. As will be seen the proofs of general theorems are often simpler than special proofs for special cases.

The first application of Wiener's method to remainders in tauberian theorems was given by Beurling in 1938. His work has been continued by Lyttkens. Among other things she has proven theorems where conditions on the Fourier transform of the kernel are required only in the upper half-plane. Such theorems are certainly important e.g. in number theory. I have chosen to formulate my theorems in a less general way in this respect, but it is easy to see that the method can be used in many cases also under the weaker assumptions.

For help in the preparation of the notes for publication thanks are due to the staff of Matscience but also to J.E.Andersson, A.Ganelius, T.O.Ganelius and K.Lidhag.

August 1971

Tord Ganelius

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