

# Lecture Notes in Mathematics

A collection of informal reports and seminars

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**Separable Algebras  
Over Commutative Rings**

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## PREFACE

This manuscript has its origin in courses given by the authors at Purdue University and Michigan State University. We were first introduced to this material by D. K. Harrison and we hope his influence is apparent. We also had access to notes prepared by G. J. Janusz for a course he gave at the University of Chicago.

We have laid the foundation for the study of the Brauer group of a commutative ring but have omitted the "spectral sequences" and the "method of descent" in the hope this will make the material more accessible. Cross-references are made as follows: Theorem 2.1 of Chapter 3 is referred to as Theorem 2.1 if the reference is in Chapter 3 and as Theorem 3.2.1 otherwise. We would like to thank the many students who have influenced and encouraged this publication, and Mrs. Glendora Milligan, who carefully typed the final form for reproduction.

September 1970

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