Lecture Notes in Mathematics

A collection of informal reports and seminars
Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

135

Wilhelm Stoll
University of Notre Dame, Notre Dame/IND/USA

Value Distribution of Holomorphic Maps into Compact Complex Manifolds

Springer-Verlag
Berlin · Heidelberg · New York 1970

PREFACE

The theory of value distribution in several complex variables received a new impetus by the results of Levine [14] and Chern [2] in 1960. In 1965, Bott and Chern [1] developed a theory of equidistribution of zeros of holomorphic sections in vector bundles. In 1967, a first main theorem of value distribution for holomorphic maps into the projective space was given in [28]. In 1968, Hirschfelder [6] was able to extend this theory to holomorphic maps into compact complex manifolds for admissible families of analytic sets parameterized by a homogeneous Kaehler manifold. Independently, at the same time, Wu [33] developed a similar theory, which treated only holomorphic maps of fiber dimension 0 into a compact Kaehler manifold for the point family.

During the Fall Quarter of 1969, the author conducted a research seminar at Stanford University, where he presented the theory, as given in these Lecture Notes. The proximity form was now constructed directly and explicitly. This made it possible to drop the homogeneity condition of Hirschfelder's approach.

The author received great help by a communication of Hirschfelder showing that Wu's proximity form is also a proximity form in the case of positive fiber dimension (Hirschfelder [7a]). This communication was received at the beginning of the seminar. Hirschfelder's observation helped the author to define the singular potential (Definition 5.1) and to show that it is a proximity form.

The theory as presented here, owes much to Hirschfelder and Wu. It gives new results as well as it represents and unifies ideas and results of Chern, Hirschfelder, Wu and the author. Proofs within the theory are given whether new or not. Outside results of other theories, as Hodge theory on Kaehler manifolds, multiplicity of holomorphic maps and the continuity of the fiber integral are used without proofs.

An exception is made with the theory of integration over the fibers of a differential or holomorphic map. The author learned of this operator from Bott and Chern [1]. The use of the operator seems to be spreading. However, no account with precise statements and complete proofs seems to have appeared. Therefore, with the encouragement of A. Andreotti and S. S. Chern, the author has given such an account in Appendix II without any claim to originality.

Appendix I contains a group of highly technical results requiring complicated proofs. A large part of this appendix consists of an almost literal reproduction of parts of §2 of Hirschfelder's thesis [6]. This is not easily accessible since much of it was suppressed in [7] due to space restrictions.

Originally, it was anticipated to include in these notes an outline of the theory of Bott and Chern [1], and to show, how their equidistribution theory in ample vector bundles can be obtained from the theory here respectively from [28]. However, this will appear at another place.

This work was done at Stanford University while the author was supported by the University of Notre Dame and by the National Science Foundation under Grant NSF GP7265. The author wishes to express his gratitude to these institutions for their help and support to make this work possible.

Wilhelm Stoll
Notre Dame and Stanford
Spring 1969

CONTENT