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Lectures on the Eilenberg-Moore Spectral Sequence

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Introduction

These notes are an outgrowth of lectures that I delivered during the spring of 1969 at Aarhus University. They represent my feeble attempts to organize in a coherent way the circle of ideas revolving about the spectral sequence introduced by Eilenberg and Moore in [18], The first part of these notes presents a new construction for the spectral sequence based on viewing it as a Kunneth spectral sequence in a suitable category, the category Top/B of spaces over a fixed space B. This idea has also been employed by L. Hodgkin whose work has not yet appeared in print. The category Top/B and its pointed analog offer a suitable setting for many other ideas, constructions and theorems. We reccomend that the interested reader consult [8] [19] [23] [28] [33] and/or [45] for further material in this direction.

The second part of these notes deals with a situation in which the Eilenberg-Moore spectral sequence has proved most tractable. Namely the study of stable Postnikov systems. Most, if not all , of the material of this section is an outgrowth of my joint published [31] and unpublished work with J.C.Moore. I have tried to collect and clarify the results that are spread through [31],[38], [39], and [44] as they apply to a particular problem, namely how does the Pontrjagin ring of a Hopf space depend on the number of k-invariants. These same ideas and techniques have proven useful in other related situations (see for example [37],[40],[41]) and it is hoped they will commend themselves to further study.

The third part of these lectures is concerned with several results that may be obtained from the precursor to the Eilenberg-Moore spectral sequence introduced by J.F. Adams in [1]. Many of the results we discuss in this part are an outgrowth of my unpublished work with Alan Clark. I believe that these results have been known to the experts for some time. They demonstrate the distinct advantage to be obtained form the algebraic approach of [1] and [18] in certain situations.

There are individual introductions to the three separate parts of these notes and we refer to them for a more detailed summary of the material covered.

I would like to express thanks to my many collaborators through the years, P.F. Baum, A. Clark, J.C. Moore and R.E. Stong, for the help and guidance that they have provided me with. I am most grateful to Professor Svend Bundgaard of the Matematisk Institut at Aarhus for his kind invitation to deliver these lectures and to the participants in the topology seminar, who helped me to clarify many obscure points. I am indebted to the Air Force Office of Scientific Research for a Post Doctoral Fellowship and to M. Leon Motchane of I.H.E.S. who provided me with a setting conducive to writting these notes.

Finally I would like to express my gratitude to my wife who typed a large portion of these notes and tolerated my foul moods during their writting and the research that led up to them.

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