

Lecture Notes in Mathematics

A collection of informal reports and seminars

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Left Principal Ideal Rings



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PREFACE

The aim of this monograph is to present a self-contained account of the structure theory and the ideal theory of principal left ideal rings with unity (pli-rings, for short). It is not an account of all aspects of pli-rings. Indeed, it contains nothing at all concerning modules over pli-rings and very little concerning factorization of elements.

A glance at the table of contents will reveal a little of what the monograph contains. Moreover, each chapter has a separate introduction. In each section, the main results are stated as soon as enough definitions are given to make the statements meaningful. This arrangement may help to get a detailed idea of the contents unhampered by proofs.

A first year graduate course in algebra covering Artinian rings and an acquaintance with ordinals would be enough to read most of the monograph.

Some parts of this monograph are taken from my thesis written under the supervision of Professor Newcomb Greenleaf. I wish to thank him for the encouragement and advice I received from him. Thanks are due to Professor R. E. Johnson who read the first draft of this monograph and made a number of useful suggestions. In particular, in §6 of Chapter II, I have followed the simple and elegant method suggested by him. I wish to thank Professor Alex Rosenberg and Professor P. J. Hilton for their interest in this monograph. I wish to thank Mrs. Manju Bewtra for pointing out some inaccuracies in the first draft.

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