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An Introduction to the Theory of Algebraic Surfaces

Notes by James Cohn, Harvard University, 1957 – 58

1969



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PREFACE

These are the lecture notes of a course which I gave at Harvard University in 1957-58. As the supply of the original mimeographed copies of these notes has been exhausted some years ago, and as there seems to be some evidence of a continued demand, I readily accepted a proposal by the Springer Verlag to republish these old notes in the current series of "Lecture Notes in Mathematics." I refrained from making any changes or revisions, for I feel that these notes can best serve their purpose if they are published in their exact original form. The purpose of these notes is to acquaint the reader with some basic facts of the theory of algebraic varieties, and to do that by self-contained, direct and I would almost say - ad hoc methods of Commutative Algebra, without overwhelming the reader with a mass of material which has a degree of generality out of all proportion to the immediate object at hand. I should also mention, incidentally, that the title of these lecture notes is somewhat misleading, for only three of the sixteen sections (namely, sections 7, 14, and 19) deal specifically with algebraic surfaces; the remaining thirteen sections deal with varieties of any dimension.

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December, 1968

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