

# Lecture Notes in Mathematics

A collection of informal reports and seminars

Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

76

---

**R. G. Swan**

University of Chicago, Chicago, Illinois

1968

**Algebraic K-Theory**

---



Springer-Verlag Berlin · Heidelberg · New York



## INTRODUCTION

These notes are taken from a course on algebraic K-theory which I gave at the University of Chicago in 1967. They also include some material from an earlier course on abelian categories, elaborating certain parts of Gabriel's thesis. The results on K-theory are mostly of a very general nature. I hope to treat some of the deeper parts of the theory, in particular, the case of finite groups, in a subsequent set of notes. The point of view taken in these notes is not always consistent. I generalized a number of results on modules to abelian categories but did not hesitate to return to modules when the going got a bit rough. Most of the material here has appeared in some form in the literature, the main exception being Chapter 14 which is based on unpublished results of Milnor, Kervaire, and Steinberg.

I would like to thank J. Burroughs, G. Evans, and M. Schacher for taking the notes. In particular, I would like to thank G. Evans who collected the notes, rewrote them all in readable form, and proofread the final version. Special thanks are due to S. Mac Lane who suggested to us the idea of publishing these notes and who arranged for their typing, which was done by M. Benson, and their publication.

## TABLE OF CONTENTS

PART I. CATEGORY THEORY	1
Quotient Categories	40
PART II. K-THEORY	66
Chapter	
1. Definition of $K_0(\underline{A})$ and Some Examples	66
2. Krull-Schmidt Theorems and Applications	75
3. Definition of $G(R)$ and Examples	92
4. The Connection Between $K_0(R)$ and $G_0(R)$	100
5. Localization and Relation Between $G_0(R)$ and $G_0(R_S)$	109
6. $K_0$ of Graded Rings	124
7. $\text{Spec}(R)$ and $H(R)$	132
8. Picard Group and the Determinant	146
9. Basic Topological Remarks	155
10. Chain Complexes and the Nilpotence of $K_0(R)$	161
11. Serre's Theorem	171
12. Cancellation Theorems	183
13. $K_1(\underline{A})$	193
14. $K_2(R)$	204
15. The Exact Sequence of $K_i$ 's	211
16. Further Results on $K_1$ and $K_0$	224
17. Relations Between Algebraic and Topological K Theory	247
BIBLIOGRAPHY	257
INDEX	258
LIST OF SYMBOLS	261