

Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

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Singular Optimal Control:
The Linear-Quadratic Problem



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PREFACE

Singular Optimal Control: The Linear-Quadratic Problem is a monograph aimed at advanced graduate students, researchers and users of singular optimal control methods. It presumes prior exposure to the standard linear-quadratic regulator problem, and a general maturity in linear systems theory.

A number of advances in singular, linear-quadratic control have taken place very recently. The book is intended to present an up-to-date account of many of these advances. At the same time, the book attempts to present a unified view of various approaches to singular optimal control, many of which are apparently unrelated.

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TABLE OF CONTENTS

I	SINGULAR LINEAR-QUADRATIC OPTIMAL CONTROL - A BROAD BRUSH PERSPECTIVE	
I.1	Problem origins	1
I.2	Historical aspects of singular linear-quadratic control	2
I.3	Objective of this book	3
I.4	Chapter outline	4
II	ROBUST LINEAR-QUADRATIC MINIMIZATION	
II.1	Introduction	8
II.2	Quadratic property of the optimal performance index	9
II.3	Initial condition results and the Riemann-Stieltjes inequality	14
II.4	Robustness in problems with end-point constraints	28
II.5	Extremal solutions of Riemann-Stieltjes inequalities	36
II.6	Summarizing remarks	38
III	LINEAR-QUADRATIC SINGULAR CONTROL: ALGORITHMS	
III.1	Introduction	41
III.2	Control space dimension reduction and a standard form	45
III.3	Vector version of Kelley transformation	47
III.4	Computation of optimal control and performance index	54
III.5	Solution via Riemann-Stieltjes inequality	57
III.6	Summarizing remarks	61
APPENDIX III.A	Dolezal's theorem	62
APPENDIX III.B	Symmetry condition	62

IV	DISCRETE-TIME LINEAR-QUADRATIC SINGULAR CONTROL AND CONSTANT DIRECTIONS	
IV.1	Introduction	67
IV.2	Linear-quadratic control in discrete- time	68
IV.3	Constant directions - basic properties	73
IV.4	Control space dimension reduction	78
IV.5	State space dimension reduction	80
IV.6	Total reduction of the problem	84
IV.7	Time-varying problems, miscellaneous points and summarizing remarks	87
	APPENDIX IV.A Definitions of coefficient matrices	90
V.	OPEN QUESTIONS	92