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Lecture Notes in Mathematics

A collection of informal reports and seminars

Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

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Seminar on Differential Equations and Dynamical Systems

Edited by G. Stephen Jones

Institute for Fluid Dynamics and Applied Mathematics
University of Maryland, College Park, Maryland

Seminar Lectures at the University of Maryland in August 1967
given by A. Cellina, G. M. Dunkel,
G. S. Jones, J. Kato, A. Strauss, J. A. Yorke, T. Yoshizawa

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PREFACE

It can be well argued that the informal working seminar is the most valuable vehicle for the bringing together of ideas in a lively research effort. Researchers and students alike profit by seeing results presented in preliminary form before most of the original motivating factors are polished away. This collection of papers was presented in such a seminar at the University of Maryland and it has been the intent to preserve the informality of original presentation as fully as possible.

Recent results in stability, control, and hereditary dependence for differential equations and geometric invariance in dynamical systems is the general theme of this collection. Many results are only sketchily presented and can be expected to be eventually published in fuller detail elsewhere. It is felt that this material can be a valuable supplement to standard texts for use in advanced topic courses in the theory of differential equations.

The editor and authors of this work would like to thank the Institute for Fluid Dynamics and Applied Mathematics of the University of Maryland for sponsoring the seminar which has resulted in this report. We are also indebted to the National Science Foundation for financial support. For an excellent job in typing this text we are indebted to Mrs. Madeleine Mills.

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