# Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

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Y. Sawaragi · T. Soeda · S. Omatu

# Modeling, Estimation, and Their Applications for Distributed Parameter Systems

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## Preface

Recent developments in estimation and control problems have concentrated primarily on systems whose dynamic behavior can be adequately described by ordinary differential equations. In view of the present trend of rapidly advancing science and technology, it is most likely that the control systems call for more stringent design specifications and more complex control objectives, particularly in industrial processes and environmental systems. This generally requires the consideration of a more accurate mathematical description of the systems to be controlled.

In general it seems that all physical systems are intrinsically distributed in nature. However, in many physical situations, the system's spatial energy distribution is sufficiently concentrated or invariant in form during the course of motion so that an approximated lumped parameter description may be adequate. On the other hand, the spatial energy distributions of many practical systems are widely dispersed. For example, we can imagine optical or acoustic waves propagating in the random media. Hence, it is desired to consider the precise estimation and control problems of certain spatially distributed parameter models which are described in terms of stochastic partial differential equations.

In this book, we treat the estimation and control problems for linear stochastic distributed parameter systems. Background material in probability theory and stochastic processes in Hilbert spaces are given in Chapter 2. Chapters 3-4 are devoted to a study of optimal estimation problems for a linear stochastic distributed parameter system and Chapter 5 presents the optimal sensor location problems. Chapter 6 develops the optimal control problems for a linear stochastic distributed parameter system.

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TABLE OF CONTENTS


| | | |
|---|---|---|
| Chapter 1. | Introduction. | 1 |
| Chapter 2. | Mathematical Preliminaries. | 3 |
| 2.1. | Probability theory in Hilbert spaces. | 3 |
| 2.1.1. | Probability theory in Euclidean spaces. | 3 |
| 2.1.2. | Probability theory in Hilbert spaces. | 9 |
| 2.1.3. | Stochastic integral in Hilbert spaces. | 16 |
| 2.1.4. | Ito's stochastic integral in Hilbert spaces. | 28 |
| 2.2. | Stochastic differential equation in Hilbert spaces. | 36 |
| 2.3. | Ito's lemma in Hilbert spaces. | 40 |
| 2.4. | Stochastic partial differential equations. | 50 |
| 2.4.1. | Deterministic partial differential equations. | 50 |
| 2.4.2. | Stochastic partial differential equations. | 59 |
| 2.5. | Abstract evolution theory. | 82 |
| 2.6. | Concluding remarks. | 87 |
| Chapter 3. | Optimal Estimation Problems for a Distributed parameter System. | 88 |
| 3.1. | Formal derivation for the optimal estimators. | 88 |
| 3.1.1. | Optimal filtering problems. | 88 |
| 3.1.2. | Optimal smoothing problems. | 104 |
| 3.1.3. | Optimal prediction problems. | 134 |
| 3.2. | Optimal estimators in Hilbert spaces. | 137 |
| 3.2.1. | Optimal filtering problems. | 137 |
| 3.2.2. | Optimal smoothing problems. | 155 |
| 3.2.3. | Optimal prediction problems. | 170 |
| 3.2.4. | Innovation processes. | 173 |

| | | |
|---|---|---|
| 3.3. | Optimal estimators by the parameter optimization technique. | 177 |
| 3.3.1. | Optimal filtering problems. | 177 |
| 3.3.2. | Optimal smoothing problems. | 184 |
| 3.3.3. | Optimal prediction problems. | 190 |
| 3.3.4. | Kernel representations of the operator equations. | 193 |
| | APPENDIX: Green's formula. | 198 |
| 3.4. | Concluding remarks. | 200 |
| Chapter 4. | Existence theorems for the optimal estimations. | 201 |
| 4.1. | Existence theorem for Riccati equation. | 201 |
| 4.2. | Existence theorems for the optimal filtering and prediction estimators. | 210 |
| 4.3. | Existence theorem for the smoothing gain function. | 211 |
| 4.4. | Concluding remarks. | 215. |
| Chapter 5. | Optimal sensor location problems. | 216 |
| 5.1. | Problem formulation. | 216 |
| 5.2. | Comparison theorem. | 221 |
| 5.3. | Existence theorem and optimality conditions. | 228 |
| 5.4. | Numerical examples. | 235 |
| 5.5. | Concluding remarks. | 242 |
| Chapter 6. | Stochastic optimal control problems. | 243 |
| 6.1. | Problem formulation. | 243 |
| 6.2. | Hamilton-Jacobi equations. | 245 |
| 6.3. | Optimal boundary control problems. | 249 |
| 6.4. | Optimal distributed control problems. | 257 |
| 6.5. | Optimal control with noisy observations. | 259 |
| 6.6. | Concluding remarks. | 263 |