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PREFACE

The momentum of the mind is all toward abstraction.

- Wallace Stevens, Opus Posthumous

Within the framework of twistor theory the structure of spacetime is relegated, in contrast to the position which it has held since the beginning of the twentieth century, to a status of secondary character. Whereas in the past spacetime has always served as the background against which phenomena are to be interpreted—and indeed, according to Einstein's theory of gravitation, spacetime serves moreover as a basic dynamical entity itself—the new view which the twistor theorists are advocating takes twistor space, with the many rich and variegated aspects of its complex analytic structure, as the primary descriptive device and dynamical construction in terms of which phenomena are to be understood.

The difficulties inherent in a spacetime description have long been appreciated by many authors. Julian Schwinger, for example, in his preface to Selected Papers on Quantum Electrodynamics summarizes the situation aptly when he remarks that "... The localization of charge with indefinite precision requires for its realization a coupling with the electromagnetic field that can obtain arbitrarily large magnitudes. The resulting appearance of divergences, and contradictions, serves to deny the basic measurement hypothesis. We conclude that a convergent theory cannot be formulated consistently within the framework of present space-time concepts. To limit the magnitude of interactions while retaining the customary coordinate description is contradictory, since no mechanism is provided for precisely localized measurements." With a similar attitude towards this question Einstein, at the end of The Meaning of Relativity, concludes that "One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality." Of course when he refers to a continuum Einstein means spacetime, taken with its usual real differentiable structure. In twistor theory, however, the continuum which arises is that of the complex number system, and those aspects of the geometry of twistor space which are of interest to physics stem more specifically from its complex analytic structure, rather than its real differentiable structure. The general characterization of the structures which can arise in the case of complex analytic manifolds has been the subject of intense investigation by mathematicians, especially with the advent of the powerful techniques of sheaf cohomology theory. One of the precepts of twistor theory is that here, within a suitably formulated sheaf cohomological framework, we have the proper basis for a "purely algebraic" description that is compatible both with the ideas of relativity and with the principles of quantum mechanics.

This view has met with a reasonable degree of success, and it has been possible, using methods of algebraic geometry and complex analytic geometry, for twistor theorists to assemble the outlines of a new approach to elementary particle physics. The subject is still in its infancy and in a rapid state of development, and thus many of its results are only of a preliminary character and are both subject to and deserving of considerable modification and improvement. In spite of their tentative nature, it seemed appropriate nonetheless to prepare an account of some of these matters for a wider audience, with the hope that it might stimulate or otherwise prove a useful aid in further and more extensive research into the subject. With this purpose in mind the following study is presented.

Although a fair amount of background material is covered in Chapters 2 and 3, the reader previously uninitiated into the mysteries of twistor theory may find it necessary to consult some additional references. For the two-component spinor formalism see Pirani (1965), Penrose (1968a), and the forthcoming book by Rindler and Penrose. For further reading in basic twistor theory see Penrose (1967), Penrose and MacCallum (1972), and Penrose (1975a). Although a specialized knowledge of elementary particle physics is not necessary, at the outset, for reading this volume, it is assumed nonetheless that the reader is familiar with basic quantum mechanics, and is acquainted already, to some extent, with the quark model.

The author is indebted to many of his colleagues for their help in the preparation and development of this material, particularly to R. Penrose who originated many of the ideas discussed here, and who has acted as a constant source of illumination and inspiration. G.A.J. Sparling has contributed extensively to this work, and the author wishes to thank him for many helpful discussions. I would also like to thank many of my colleagues at Oxford and elsewhere, including D.M. Blasius, M. Eastwood, M.L. Ginsberg, A. Hodges, S.A. Huggett, T.R. Hurd, R. Jozsa, E.T. Newman, A. Popovich, Z. Perjés, I. Robinson, M. Sheppard, L. Smarr, P. Sommers, K.P. Tod, Tsou S.T., M. Walker, R.S. Ward, and N.M.J. Woodhouse, for useful conversations and suggestions related to the work described herein. The author is grateful to B.S. DeWitt, C.M. DeWitt, R. Matzner, L. Shepley, H.J. Smith, and the late Alfred Schild, as well as other colleagues at the University of Texas at Austin, for their hospitality shown during the author's 1974 visit, when some of the ideas preliminary to the material described here were worked out. The author has profited much from his regular visits, supported by the Clark Foundation, to the University of Texas at Dallas, and he would like to thank I. Ozsvath, W. Rindler, I. Robinson, and J.R. Robinson for their hospitality. Likewise the author has benefited from his visits to the Astronomy Department at the University of Virginia, and gratitude is expressed to W. Saslaw, and other colleagues there, for their hospitality. I am grateful to J. Ehlers, M.L. Ginsberg, C.J. Isham, R. Penrose, G.A.J. Sparling, and N.M.J. Woodhouse for reading earlier drafts of the manuscript and contributing many corrections and helpful suggestions for improvement.

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This volume is dedicated to my mother and my father.

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