

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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Yoshimi Saitō

Spectral Representations  
for Schrödinger Operators  
With Long-Range Potentials

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Springer-Verlag  
Berlin Heidelberg New York 1979

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AMS Subject Classifications (1970): 35 J10, 35 P25, 47 A40

ISBN 3-540-09514-4 Springer-Verlag Berlin Heidelberg New York  
ISBN 0-387-09514-4 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data

Saitō, Yoshimi. Spectral representations for Schrödinger operators with long-range potentials. (Lecture notes in mathematics ; 727) Bibliography: p. Includes index. 1. Differential equations. Elliptic. 2. Schrödinger operator. 3. Scattering (Mathematics) 4. Spectral theory (Mathematics) I. Title. II. Series: Lecture notes in mathematics (Berlin) : 727. QA3.L28 no. 727 [QA377] 510'.8s [515'.353] 79-15958

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Printed in Germany

Printing and binding: Boltz Offsetdruck, Hemsbach/Bergstr.  
2141/3140-543210

## PREFACE

The present lecture notes are based on the lectures given at the University of Utah during the fall quarter of 1978. The main purpose of the lectures was to present a complete and self-contained exposition of the spectral representation theory for Schrödinger operators with long-range potentials.

I would like to thank Professor Calvin H. Wilcox of the University of Utah for not only the opportunity to present these lectures but also his unceasing encouragement and helpful suggestions. I would like to thank Professor Willi Jäger of the University of Heidelberg, too, for useful discussions during the lectures.

Yoshimi Saitō

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