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David K. Haley

Equational Compactness in Rings

With Applications to
the Theory of Topological Rings



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