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FOREWORD

I met Melvin Scott at the SIAM meeting in San Francisco, December 1975. In discussing the beauty of the host city and other finer points, we reached a mutual decision that there was need for a conference, workshop, and/or meeting with a central theme of Working Codes for Boundary-Value Problems in ODEs. Persuant to this conversation and some prodding telephone calls from Mel, we selected the organizing committee, we all agreed to serve, and we all worked to perform the many tasks necessary. The organizing committee of:

Jim Daniel - University of Texas at Austin
Gene Denman - University of Houston
Paul Nelson - Texas Tech University
(Paul was at Georgia Tech 1977-78)
Melvin Scott - Sandia Laboratories

was not selected because we felt that all knowledge springs from the Southwest, but because we had no budget for planning and we were close, geographically. We gratefully acknowledge our employing institutions for the individual assistance given during the planning.

We met in Austin, Texas to outline the proposals for support, identify codes for demonstration and to be the subject of expository papers, identify possible conference dates, compile mailing lists for solicition of participants, and discuss guidelines of selecting the final participants giving papers. I am especially pleased that the conference was well received and believe that this was largely due to the open-minded professional attitudes of the organizing committee at this meeting and through their subsequent efforts.

Our primary motivation came from appreciation of the advanced state of codes in the area, the fact that papers describing codes are often not publishable in the usual journals, that much was to be gained by having code authors, users, and potential users together in a pleasant surroundings, and that prompt publication of a proceedings would be proper.

We are grateful to Editors Juris Hartmanis and Cleve Moler for accepting this Proceedings into their outstanding series. The Proceedings contain many valuable papers and we express our appreciation to the authors for their efforts and also thank the participants who were not authors for their excellent participation.

The Working Conference on Codes for Boundary-Value Problems in Ordinary Differential Equation was held May 14-17, 1978 at the University of Houston. This meeting was jointly supported by the National Science Foundation, Grant No. MCS 77-22818 and the Department of Energy, Grant No. ER-78-G-05-5885. We are especially grateful for their support.

These proceedings contain the invited papers and the contributed papers presented at the Conference. Various workshops to discuss working codes, algorithms utilized in the codes and benchmark problems were held, and workshop reports are given in these Proceedings.

The first three papers in the Proceedings are invited survey papers on numerical methods for two-point boundary-value problems, initial value integrators, and nonlinear equation solvers. Three additional surveys are interspersed in the Proceedings, two on methods that are not as widely understood as finite difference and shooting methods, namely projection methods and invariant imbedding. The third survey paper describes various approaches in finite-difference and collocation codes for mesh selection.

Invited papers on working codes that are available for problem-solving are given in these proceedings. The codes presented at the conference were a) a shooting code for Sturm-Liouville problems (Bailey-Shampine), b) a finite-difference code for first-order systems (Pereyra), c) a multiple shooting code for first-order systems (Bulirsch), d) a multiple shooting code with orthogonalization (Scott), e) a spline-collocation code for mixed-order systems (Asher, Christiansen and Russell), and f) a shooting code for multipoint boundary-value problems arising in system identification (Childs-Porter). All of the above codes were demonstrated in some manner at the meeting and information on the codes is available from the authors or other sources.

The set of codes available from the NAG Library in England was described by Gladwell but were not demonstrated at the conference.

Numerous contributed papers were presented and the papers are published in these Proceedings. These papers describe particular aspects of solving boundary-value problems or discuss results of a numerical experiment in which boundary-value problems are encountered.

The code demonstration workshop at the conference was a valuable part of the meeting in that the workshop acted as a stimulus for active discussion of codes. Information gained from implementation of codes in a "conference environment" should be of value for future conferences. Code transportability did not prove to be a major problem, although considerable pre-conference effort is required for successful implementation.

We did exercise editorial license as a committee. Some worthwhile papers were not accepted for presentation at the conference. We have made some minor changes in some manuscripts to enhance clarity and have requested rewriting of others to meet the conference goals. I accept the responsibility for any errors we have made in these efforts. The statements and claims of the individual authors are their own and do not necessarily reflect the views of the committee, our institutions, or the sponsoring agencies NSF and DOE.

Finally, I wish to express my most sincere appreciation to Gene and Norma Denman and the University of Houston for the excellent facilities and hospitality they provided. As several participants have written, I owe the success and pleasant atmosphere of the conference to my wife Shirley for her many hours of work in making reservations, calling taxis, being a pleasant guide, being a stern camp director, and being herself. Other than the thanks I have spoken, the auditors will let me thank her only by dedicating this work to her and our wonderful daughter, Meredith.

S. Bart Childs
August 1978
College Station, Texas

Those interested in obtaining a code should write the code authors directly. See the Workshop Summary beginning on page 370.

Several of the papers do not have individual bibliography. A combined bibliography of 290 entries is at the end of these Proceedings. I intend to update this bibliography and distribute it on some logical basis. Interested parties are encouraged to send relevant entries to me for inclusion in the next release of the bibliography.

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