Part II

Algorithms and Applications of Advanced Grid Technology

The current part of the monograph including Chaps. 5–7 gives an account of the geometrization of the popular comprehensive grid methods and presents an important extension of the methods, related to the application of the theory of Riemannian manifolds to the formulation of grid equations and grid functionals.

As comprehensive tools for generating grids there are chosen the operators of Beltrami and diffusion in an arbitrary metric. These operators give rise to the formulation of Beltrami and diffusion equations, respectivelly. It is established in Chap. 5 that an arbitrary one-to-one, smooth multidimensional transformation deriving a numerical grid in a domain or on a surface by the mapping approach is realized by the solution of the system of Beltrami equations in a suitable monitor metric specified in the physical geometry. The system can be interpreted as the multidimensional equidistribution principle in which the monitor metric tensor is an extension of a scalar-valued weight function. While the required grid properties are realized through the specification of suitable monitor metrics that can be formulated as a combination of metric tensors with weights, each of which is responsible for providing one individual grid property.

Taking advantage of the tensor relations, the Beltrami and diffusion equations are converted in Chap. 6 into compact forms convenient for the numerical treatment by the available algorithms. Some of these forms of equations rule out division by the Jacobian of intermediate transformations. So solutions of these equations allow one to use singular intermediate transformations as demonstrated in Chap. 7. Relations of the mean curvature of the monitor surfaces to the equations for grid generation are also exhibited in Chap. 6. In this chapter the technique of the multidimensional differential geometry is also applied to the analysis of the qualitative behavior of the grids generated by the mapping approach. For this purpose a new characteristic of grid clustering is formulated. A relation between this grid measure and some geometric characteristics of grid hypersurfaces and the monitor functions forming the monitor metric is established. The well-known results for grids generated by the inverted Laplace equations about node clustering near concave boundary segments of domains and node rarefaction near convex ones are extended, using the relation, to arbitrary boundary segments and to more general elliptic equations formulated for generating grids. On the basis of the formula the monitor functions are readily estimated in the diffusion and Beltrami elliptic models of grid equations to provide grid clustering or, if it is reasonable, grid rarefaction near arbitrary segments of physical geometries.

Chap. 7 gives a description of numerical codes for generating grids with the help of the elliptic grid equations and energy functionals obtained in Chaps. 5 and 6 by changing mutually dependent and independent variables in the basic grid models, formulated in Chap. 5. The application of the grid technology advocated in the book to some gas-dynamics problems is also discussed in Chap. 7.