

Part I

Geometric Background to Grid Technology

The science of differential geometry is a powerful and helpful tool capable for significantly boosting the development of grid technology. In particular, many notions and characteristics of two-dimensional surfaces bounding physical domains such as metric tensors, their invariants, the first and second fundamental forms, curvatures and torsions of lines, mean and Gauss curvatures of surfaces, and Christoffel symbols have already been reviewed in the monographs of Warsi (1981), Thompson, Warsi, and Mastin (1985), Knupp and Steinberg (1993), and Liseikin (1999) as natural elements in defining grid quality measures and formulating appropriate variational and differential grid techniques in a unified manner regardless of the geometry of domains and surfaces.

The papers of Dvinski (1991) and Liseikin (1991) gave rise to the implementation of more general geometric objects such as multidimensional regular surfaces and Riemannian manifolds into two- and three-dimensional adaptive grid generation techniques. The known relations and techniques of the multidimensional differential geometry also present a useful tool for transforming and modernizing the physical and grid equations into a suitable form.

This part of the book, containing Chaps. 1–4, gives an introduction to the geometric and tensor theories necessary in advanced grid technology. Chapter 1 acquaints the reader with the most general aspects of grid generation. It expounds the most popular approaches and mapping techniques that have been developed for generating numerical grids in domains and on surfaces. The elementary theories of curves, multidimensional regular surfaces, and Riemannian manifolds are reviewed in Chaps. 2–4. These geometries play a crucial role in formulating comprehensive grid equations and in controlling grid properties in domains and on their boundaries. The metric tensors and basic geometry characteristics, in particular, the mean curvature of n -dimensional surfaces and Riemannian manifolds necessary for grid technology are discussed in detail.