

James J. Buckley

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Fuzzy Probability and Statistics

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James J. Buckley

# Fuzzy Probability and Statistics

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To Julianne and Helen

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