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Kang-Sin Choi Jih E. Kim

Quarks and Leptons From Orbifolded Superstring

 Springer

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Preface

Using the successful standard model of particle physics but without clear guidance beyond it, it is a difficult task to write a physics book beyond the standard model from a phenomenological point of view. At present, there is no major convincing inner space related experimental evidence against the standard model. The neutrino oscillation phenomena can be considered part of it by including a singlet field in the spectrum. Only the outer space observations on matter asymmetry, dark matter, and dark energy hint at the phenomenological need for an extension; however, there has been theoretical need for almost three decades, chiefly because of the gauge hierarchy problem in the standard model.

Thus, it seems that going beyond the standard model hinges on the desirability of resolving the hierarchy problem. At the field theory level, it is fair to say that the hierarchy problem is not as desperate as the nonrenormalizability problem present in the old V–A theory of weak interactions on the road to the standard model. An extension beyond the standard model can easily be ruled out as witnessed in the case of technicolor. However, a consistent framework with supersymmetry for a resolution of the hierarchy problem has been around for a long time. Even its culprit “superstring” has been around for twenty years, and the most remarkable thing about this supersymmetric extension is that it is still alive. So the time is ripe for phenomenologists to become acquainted with superstring and its contribution toward the minimal supersymmetric standard model in four space-time dimensions.

This book is a journey toward the minimal supersymmetric standard model (MSSM) down the orbifold road. After some field theoretic orbifold attempts in recent years, there has been renewed interest in the physics of string orbifolds and it is time to revisit them. In this book, we take the viewpoint that the chirality of matter fermions is essential toward revealing the secrets of Nature. Certainly, orbifolds are an easy way to get the chirality from higher dimensions.

Strings and their orbifold compactification are presented for the interests of phenomenologists, sacrificing mathematical rigor. They are presented in

such a way that an orbifold model can be constructed by applying the rules included here. At the end of Chap. 10, we construct a \mathbf{Z}_{12} orbifold which contains all imaginable complications. Also, we attempt to correct any incompleteness in the rules presented before in the existing literature. In the final chapter we tabulate the simplest and most widely used orbifold \mathbf{Z}_3 with $\mathcal{N} = 1$ supersymmetry, completely in the phenomenological sense of obtaining three families. These tables encompass all noteworthy models available with two Wilson lines. Since three Wilson line \mathbf{Z}_3 orbifolds do not automatically give three families, in a practical manner these tables close a chapter on \mathbf{Z}_3 orbifolds.

This book is not as introductory as a textbook, nor is it as special as a review article on a superstring topic. Instead, we aimed at an interim region so that a phenomenologist can read and directly commence building an orbifold model.

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Kang-Sin Choi
Jihn E. Kim

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