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Dynamics of Extended Celestial Bodies and Rings

 Springer

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Editor's Preface

About the Dynamics of Extended Bodies and of the Rings

This book is mainly devoted to celestial mechanics. Under the title above we designate the study of celestial bodies that are not considered as point-masses, as they are often in celestial mechanics, in particular, when dealing with orbital motions. On the contrary, we present and analyse in full details the recent theoretical investigations and observational data related to the effects of the extended shapes of celestial bodies.

Some basic explanations concerning the rotation of an extended body are presented as a tutorial. Then, a large position is reserved for the Earth, which obviously is the most studied planet. We find detailed explanations of the internal structure of our planet, for example, the solid crust, the elastic mantle, the liquid outer core, and the solid inner core. The equations governing its rotational and internal motions under various assumptions (presence of layers, hydrostatic equilibrium etc.) are explained, as well as the modelling of its gravity field and its temporal variations.

We also present the recent developments concerning the dynamics of various celestial bodies. Some of them, the Moon and Mercury, are subject to complex rotational motions related to librations, which are explained exhaustively. Other celestial bodies, such as the asteroids, are undergoing permanent investigations concerning the comparisons between observational data, as light curves, and theoretical modeling of their rotation. The dynamics of these small planets considered as non-rigid bodies are explained in detail.

We also make a complete review of the effects of the impacts on planets and asteroids, and more precisely on their rotational and orbital characteristics. The earlier studies concerning this topic the subject of intensive research are presented.

The concluding part of this book is devoted to the dynamics of the rings and a detailed account of the various equations that govern their motions and evolutions.

We hope that this book will serve as a basis for anybody who wants to become accustomed with the dynamics of extended bodies, and also to get the relevant bibliographic background.

The Thematic School of the CNRS at Lanslevillard

This book is the result of a Thematic School organized by the CNRS (Centre National de la Recherche Scientifique) at Lanslevillard (French Alps) in March 2003, in continuation of previous Winter Schools of Astronomy, organized by C. Froeschlé and his colleagues. This school gathered about fifty people interested in the epistemology, as well as the recent developments in the fields of the rotation of celestial bodies (such as planets and asteroids) and of the rings (such as one around Saturn). This school was organized with the financial support of the CNRS by the intermediary of the “formation permanente” (continuing formation).

We are very grateful to Victoria Terziyan, responsible for the Thematic Schools at CNRS, who was deeply involved in the management of the school, as well as to Liliane Garin and Teddy Carlucci (SYRTE, Observatoire de Paris) who were responsible for the organisation.

Observatoire de Paris
November 2005

Jean Souchay

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