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B. Bandyopadhyay · S. Janardhanan

Discrete-time Sliding Mode Control

A Multirate Output Feedback Approach

With 68 Figures

Springer

Authors

Prof. B. Bandyopadhyay
S. Janardhanan

Interdisciplinary Programme in
Systems and Control Engineering
Mumbai-400 076
India

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Mātr Devo Bhavāh,
Pitr Devo Bhavāh,
Āchārya Devo Bhavāh,

(Mother is a diety, Father is a diety, Teacher is a diety)

Dedicated to our parents and teachers who made us capable enough to write this book

Preface

Sliding mode control is a simple and yet robust control technique. In case of sliding mode control, the system states are made to confine to a selected subset of the state space so as to achieve some desirable dynamics. Traditionally, a relay-based control has been used for this purpose and had its roots in the variable structure system philosophy. Developed in the erstwhile Soviet Union, the concept was pioneered by Vadim Utkin.

With the incerasing use of computers and discrete-time samplers in controller implementation in the recent past, discrete-time systems and computer based control have become topics that have a lot of potential in them. This had opened up the field of sliding mode control of discrete-time systems. Many researchers; W. B. Gao, E. Misawa, A. Bartoszewicz, K. Furuta, C. Milosavljevic, to cite a few had worked in this field. However, much of the work had been concentrated on state feedback based control. But, it is of common knowledge that only the system output is available for the controller design. More often than not, the system output is not coincident with the system state. This leads to the requirement of output feedback based sliding mode control strategies.

The existing literature on output feedback sliding mode control is either very restrictive, by being applicable to only a specific class of systems, even when one looks at the control of LTI systems alone. A wider class of systems can be controlled, if one adopts dynamic sliding mode controllers. However, the system complexity is increased in the process.

This is the motivation of this monograph : An output feedback sliding mode control philosophy which can be applied to almost all controllable and observable systems, while at the same time being simple enough as not to tax the computer too much. We found the answer in the synergy of the multirate output sampling concept and the concept of discrete-time sliding mode control.

This work would have been incomplete had it not been for the kind cooperation and help from many. Particularly, we needed much help from our associates Vishvjit K. Thakar, Vitthal S. Bandal and T. C. Manjunath in find-

ing appropriate applications to complete the final chapter of the monograph. We would like to use the oppurtunity to thank them.

Mumbai, *Bijnan Bandyopadhyay*
May 2005 *Janardhanan Sivaramakrishnan*

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Nomenclature

	Abbreviations
DSM	Discrete-time Sliding Mode
DSMC	Discrete-time Sliding Mode Control
DTSM	Discrete-time Terminal Sliding Mode
LTI	Linear Time-Invariant
MIMO	Multi Input Multi Output
MROF	Multirate Output Feedback
QSM	Quasi-Sliding Mode
QSMB	Quasi-Sliding Mode Band
QSMC	Quasi-Sliding Mode Control
RP	Representative Point
SISO	Single Input single output
SMC	Sliding Mode Control
TSM	Terminal Sliding Mode
VSS	Variable Structure System
	List of Symbols
$(\bullet)^T$	General notation for the matrix transpose operation
$\mathbb{R}$	The field of real numbers
$\mathbb{R}^n$	The vector space of vectors of length l with real entries
$\tilde{d}(k)$	Disturbance vector in an LTI system
A_0	State matrix in continuous-time model of time delay LTI system
A_1	Delayed-State matrix in continuous-time model of time delay LTI system
B_0	Input matrix in continuous-time model of time delay LTI system
B_1	Delayed-Input matrix in continuous-time model of time delay LTI system
C	Output matrix of discrete-time LTI model
c^T	Sliding function parameter
c_t^T	Terminal sliding function parameter
e_x	Error between system and reference state vectors
G_0	State matrix in discrete-time model of LTI system with time delay

G_1	Delayed-state matrix in discrete-time model of LTI system with time delay
H_0	Input matrix in discrete-time model of LTI system with time delay
H_1	Delayed-input matrix in discrete-time model of LTI system with time delay
$I_d(\bullet)$	Identity function
k_u	Ratio of input delay to input sampling time
k_x	Ratio of state delay to input sampling time
k_y	Ratio of output delay to input sampling time
m	Number of inputs of an LTI system model
N	Ratio of input and output sampling rates in a multirate system
n	Number of states of an LTI system model
p	Number of outputs of an LTI system model
r	Reference signal
s	Sliding function
T	General notation for a $n \times n$ transformation matrix
T_{pre}	Reference preview time of tracking controller for nonminumum phase system
u	Control input of a dynamical system
V	Lyapunov function
x	System state of a dynamical system
y	System output of a dynamical system
y_k	Multirate sampled system output
ш	Shuffle product operator
δ	Width of the quasi-sliding mode band
Γ	Input matrix of discrete-time LTI model sampled at interval of Δ sec
Γ_τ	Input matrix of discrete-time LTI model sampled at interval of τ sec
ζ_d	Sliding sector in a discrete-time LTI system
Φ	System state matrix of discrete-time LTI model sampled at interval of Δ sec
Φ_τ	System state matrix of discrete-time LTI model sampled at interval of τ sec
Σ	Nonlinear system representation in continuous time
τ_u	Delay in input in LTI system with input delay
τ_x	Delay in state in LTI system with state delay
τ_y	Delay in output in LTI system with output delay