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Quantum Field Theory and Noncommutative Geometry

Springer

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Preface

This book is based on the workshop "Quantum Field Theory and Noncommutative Geometry" held in November 2002 at Tohoku University, Sendai, Japan. This workshop was the third in a series, the first one having been held at the Shonan International Village at Hayama in Kanagawa-ken in 1999, and the second one at Keio University, Yokohama in 2001. The main aim of these meetings is to enhance the discussion and cooperation between mathematicians and physicists working on various problems in deformation quantization, noncommutative geometry and related fields.

The workshop held in Sendai was focused on the topics of noncommutative geometry and an algebraic approaches to quantum field theory, which includes the deformation quantization, symplectic geometry and applications to physics as well as topological field theories.

The idea to treat quantized theories by using an algebraic language can be traced back to the early days of quantum mechanics, when Heisenberg, Born and Jordan formulated quantum theory in terms of matrices (matrix mechanics). Since then, a continuous effort has been made to develop an algebraic language and tools which would also allow the inclusion gravity. Among the physicist is point of view, the concept of a minimum length is discussed many times in various theories, especially in the theories of quantum gravity. Since the string is an extended object, string theory strongly suggests the existence of a minimum length, and this brought the discussion on the quantization of space into this field. However, this discussion raised several problems, in particular, how such a geometry with minimum length should be formulated and how a quantization should be performed in a systematic way.

A hint in this direction came from the theory of quantum groups, which had been developed in the 1980s and which gave a method to deform an algebra to become noncommutative, thereby preserving its symmetry as a q-deformed structure. Nearly at the same time A. Connes published his work on noncommutative differential geometry. It was the impact from these two new fields, that put forward the research on quantized spaces, and drew more and more the physicists' attention towards this field.

Noncommutative differential geometry (NCDG) led to striking extensions of the Atiyah-Singer index theorem and it also shows several common points

with deformation quantization. Another result is the development of noncommutative gauge theory, which became a very promising candidate as an the effective theory of the so-called D-brane; a D-brane is a configuration which evolved in the course of the development of string theory, leading to solutions of nonperturbative configurations of the string in the D-brane background. Inspired by the possibilities opened by NCDG; there is now a number of physicists developing the "matrix theory", about 80 years after the "matrix mechanics".

Deformation quantization is a quantization scheme which has been introduced by Bayen, Flato, Fronsdal, Lichnerowicz and Sternheimer. In this approach the algebras of quantum observables are defined by a formal deformation of the classical observables as formal power series. The expansion parameter is $\hbar$ and the product of these deformed algebras is the star product. Symplectic geometry and Poisson geometry fit very well to this quantization scheme since they possess a Poisson structure, and thus deformation quantization is regarded as a quantization from an algebraic point of view. As we know from the theorem of Gel'fand and Naimark, we can often realize a classical space from a suitable algebra of the classical observables. From this point of view, we expect the deformation quantization may give a reasonable quantum space, whose investigation will contribute a development to noncommutative geometry.

We collected here the lectures and talks presented in the meeting. When preparing this proceedings we made effort to make this book interesting for a wider community of readers. Therefore, the introductions to the lectures and talks are more detailed than in the workshop. Also some derivations of results are given more explicitly than in the original lecture, such that this volume becomes accessible to researchers and graduate students who did not join the workshop. A large number of contributions are devoted to presentations of new results which have not appeared previously in professional journals, or to comprehensive reviews (including an original part) of recent developments in those topics.

Now we would like to thank all speakers for their continuous effort to prepare these articles. Also we would like to thank all participants of the workshop for sticking together until the end of the last talk, thus creating a good atmosphere and the basis for many fruitful discussions during this workshop. We also greatly acknowledge the Ministry of Education, Culture, Sports, Science and Technology, Japan, who supported this workshop by a Grant-in-Aid for Scientific Research (No. 13135202).

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