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J.-R. Chazottes B. Fernandez

# Dynamics of Coupled Map Lattices and of Related Spatially Extended Systems

 Springer

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## Preface

This book assembles the notes that have been written for the lectures delivered at the school-forum CML2004, a meeting held at Institut Henri Poincaré (Paris) from the 21st of June to the 2nd of July 2004.

The purposes of CML2004 were to give a survey of the most significant achievements in the theory of coupled map lattices (CML) and of related spatially extended systems, and to stimulate debates on open problems and future directions of research.

The panorama of CML theory has been presented through a series of 15 lectures given by specialists. Additional results have been given in the form of short communications, of posters and of young scientist sessions (organised by Pierre Guiraud and Arnaud Meyroneinc). In addition to these expository sessions, stimulated discussions and public debates on questions and on relevant open problems have taken place.

About 60 participants attended the meeting. They essentially consisted of physicists and mathematicians in nonlinear dynamics, from young scientists (about 20 participants where PhD students or postdocs) to senior scientists, working either on CML or on other subjects (about 20 participants).

As *Ecole Thématique*, CML2004 received essential support from the CNRS without which the meeting could not have been realised. The following institutions also contributed to the financial support: *Ministère de la Recherche et des Nouvelles Technologies*, *Centre de Physique Théorique de Palaiseau*, *Centre de Physique Théorique de Marseille*, *École Polytechnique*, *Ministère des Affaires Étrangères*, *Université de Provence*, *École doctorale de Marseille "Physique et Sciences de la Matière"*, *European Physical Society (Young Physicist Fund)*.

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Paris, Marseille  
November 2004

*Jean-René Chazottes*  
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