

# Mechanics and Strength of Materials

Vitor Dias da Silva

---

# Mechanics and Strength of Materials

Vitor Dias da Silva

Department of Civil Engineering  
Faculty of Science & Technology  
University of Coimbra  
Polo II da Universidade - Pinhal de Marrocos  
3030-290 Coimbra  
Portugal  
E-mail: vdsilva@dec.uc.pt

Library of Congress Control Number: 2005932746

ISBN-10 3-540-25131-6 Springer Berlin Heidelberg New York

ISBN-13 978-3-540-25131-6 Springer Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable for prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media  
springeronline.com

© Springer-Verlag Berlin Heidelberg 2006  
Printed in The Netherlands

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: by the author and TechBooks using a Springer L<sup>A</sup>T<sub>E</sub>X and T<sub>E</sub>X macro package

Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper      SPIN: 10996904      89/TechBooks      5 4 3 2 1 0

---

## Preface To The English Edition

The first English edition of this book corresponds to the third Portuguese edition. Since the translation has been done by the author, a complete review of the text has been carried out simultaneously. As a result, small improvements have been made, especially by explaining the introductory parts of some Chapters and sections in more detail.

The Portuguese academic environment has distinguished this book, since its first edition, with an excellent level of acceptance. In fact, only a small fraction of the copies published has been absorbed by the school for which it was originally designed – the Department of Civil Engineering of the University of Coimbra. This fact justifies the continuous effort made by the author to improve and complement its contents, and, indeed, requires it of him. Thus, the 423 pages of the first Portuguese edition have now grown to 478 in the present version. This increment is due to the inclusion of more solved and proposed exercises and also of additional subjects, such as an introduction to the fatigue failure of materials, an analysis of torsion of circular cross-sections in the elasto-plastic regime, an introduction to the study of the effect of the plastification of deformable elements of a structure on its post-critical behaviour, and a demonstration of the theorem of virtual forces.

The author would like to thank all the colleagues and students of Engineering who have used the first two Portuguese editions for their comments about the text and for their help in the detection of misprints. This has greatly contributed to improving the quality and the precision of the explanations.

The author also thanks Springer-Verlag for agreeing to publish this book and also for their kind cooperation in the whole publishing process.

Coimbra  
March 2005

*V. Dias da Silva*

---

## Preface to the First Portuguese Edition

The motivation for writing this book came from an awareness of the lack of a treatise, written in European Portuguese, which contains the theoretical material taught in the disciplines of the Mechanics of Solid Materials and the Strength of Materials, and explained with a degree of depth appropriate to Engineering courses in Portuguese universities, with special reference to the University of Coimbra. In fact, this book is the result of the theoretical texts and exercises prepared and improved on by the author between 1989-94, for the disciplines of Applied Mechanics II (Introduction to the Mechanics of Materials) and Strength of Materials, taught by the author in the Civil Engineering course and also in the Geological Engineering, Materials Engineering and Architecture courses at the University of Coimbra.

A physical approach has been favoured when explaining topics, sometimes rejecting the more elaborate mathematical formulations, since the physical understanding of the phenomena is of crucial importance for the student of Engineering. In fact, in this way, we are able to develop in future Engineers the intuition which will allow them, in their professional activity, to recognize the difference between a bad and a good structural solution more readily and rapidly.

The book is divided into two parts. In the first one the Mechanics of Materials is introduced on the basis of Continuum Mechanics, while the second one deals with basic concepts about the behaviour of materials and structures, as well as the Theory of Slender Members, in the form which is usually called Strength of Materials.

The introduction to the Mechanics of Materials is described in the first four chapters. The first chapter has an introductory character and explains fundamental physical notions, such as continuity and rheological behaviour. It also explains why the topics that compose Solid Continuum Mechanics are divided into three chapters: the stress theory, the strain theory and the constitutive law. The second chapter contains the stress theory. This theory is expounded almost exclusively by exploring the balance conditions inside the body, gradually introducing the mathematical notion of tensor. As this notion

is also used in the theory of strain, which is dealt with in the third chapter, the explanation of this theory may be restricted to the essential physical aspects of the deformation, since the merely tensorial conclusions may be drawn by analogy with the stress tensor. In this chapter, the physical approach adopted allows the introduction of notions whose mathematical description would be too complex and lengthy to be included in an elementary book. The finite strains and the integral conditions of compatibility in multiply-connected bodies are examples of such notions. In the fourth chapter the basic phenomena which determine the relations between stresses and strains are explained with the help of physical models, and the constitutive laws in the simplest three-dimensional cases are deduced. The most usual theories for predicting the yielding and rupture of isotropic materials complete the chapter on the constitutive law of materials.

In the remaining chapters, the topics traditionally included in the Strength of Materials discipline are expounded. Chapter five describes the basic notions and general principles which are needed for the analysis and safety evaluation of structures. Chapters six to eleven contain the theory of slender members. The way this is explained is innovative in some aspects. As an example, an alternative Lagrangian formulation for the computation of displacements caused by bending, and the analysis of the error introduced by the assumption of infinitesimal rotations when the usual methods are applied to problems where the rotations are not small, may be mentioned. The comparison of the usual methods for computing the deflections caused by the shear force, clarifying some confusion in the traditional literature about the way as this deformation should be computed, is another example. Chapter twelve contains theorems about the energy associated with the deformation of solid bodies with applications to framed structures. This chapter includes a physical demonstration of the theorems of virtual displacements and virtual forces, based on considerations of energy conservation, instead of these theorems being presented without demonstration, as is usual in books on the Strength of Materials and Structural Analysis, or else with a lengthy mathematical demonstration.

Although this book is the result of the author working practically alone, including the typesetting and the pictures (which were drawn using a self-developed computer program), the author must nevertheless acknowledge the important contribution of his former students of Strength of Materials for their help in identifying parts in the texts that preceded this treatise that were not as clear as they might be, allowing their gradual improvement. The author must also thank Rui Cardoso for his meticulous work on the search for misprints and for the resolution of proposed exercises, and other colleagues, especially Rogério Martins of the University of Porto, for their comments on the preceding texts and for their encouragement for the laborious task of writing a technical book.

This book is also a belated tribute to the great Engineer and designer of large dams, Professor Joaquim Laginha Serafim, who the Civil Engineering Department of the University of Coimbra had the honour to have as Professor

of Strength of Materials. It is to him that the author owes the first and most determined encouragement for the preparation of a book on this subject.

Coimbra  
July 1995

*V. Dias da Silva*

---

# Contents

---

## Part I Introduction to the Mechanics of Materials

---

<b>I</b>	<b>Introduction</b> . . . . .	3
I.1	General Considerations . . . . .	3
I.2	Fundamental Definitions . . . . .	4
I.3	Subdivisions of the Mechanics of Materials . . . . .	6
<b>II</b>	<b>The Stress Tensor</b> . . . . .	9
II.1	Introduction . . . . .	9
II.2	General Considerations . . . . .	9
II.3	Equilibrium Conditions . . . . .	12
II.3.a	Equilibrium in the Interior of the Body . . . . .	12
II.3.b	Equilibrium at the Boundary . . . . .	15
II.4	Stresses in an Inclined Facet . . . . .	16
II.5	Transposition of the Reference Axes . . . . .	17
II.6	Principal Stresses and Principal Directions . . . . .	19
II.6.a	The Roots of the Characteristic Equation . . . . .	21
II.6.b	Orthogonality of the Principal Directions . . . . .	22
II.6.c	Lamé's Ellipsoid . . . . .	22
II.7	Isotropic and Deviatoric Components of the Stress Tensor . . . . .	24
II.8	Octahedral Stresses . . . . .	25
II.9	Two-Dimensional Analysis of the Stress Tensor . . . . .	27
II.9.a	Introduction . . . . .	27
II.9.b	Stresses on an Inclined Facet . . . . .	28
II.9.c	Principal Stresses and Directions . . . . .	29
II.9.d	Mohr's Circle . . . . .	31
II.10	Three-Dimensional Mohr's Circles . . . . .	33
II.11	Conclusions . . . . .	36
II.12	Examples and Exercises . . . . .	37



<b>III</b>	<b>The Strain Tensor</b> .....	41
III.1	Introduction .....	41
III.2	General Considerations .....	41
III.3	Components of the Strain Tensor .....	44
III.4	Pure Deformation and Rigid Body Motion .....	49
III.5	Equations of Compatibility .....	51
III.6	Deformation in an Arbitrary Direction .....	54
III.7	Volumetric Strain .....	58
III.8	Two-Dimensional Analysis of the Strain Tensor .....	59
	III.8.a Introduction .....	59
	III.8.b Components of the Strain Tensor .....	60
	III.8.c Strain in an Arbitrary Direction .....	60
III.9	Conclusions .....	63
III.10	Examples and Exercises .....	64
<b>IV</b>	<b>Constitutive Law</b> .....	67
IV.1	Introduction .....	67
IV.2	General Considerations .....	67
IV.3	Ideal Rheological Behaviour – Physical Models .....	69
IV.4	Generalized Hooke’s Law .....	75
	IV.4.a Introduction .....	75
	IV.4.b Isotropic Materials .....	75
	IV.4.c Monotropic Materials .....	80
	IV.4.d Orthotropic Materials .....	82
	IV.4.e Isotropic Material with Linear Visco-Elastic Behaviour .....	83
IV.5	Newtonian Liquid .....	84
IV.6	Deformation Energy .....	86
	IV.6.a General Considerations .....	86
	IV.6.b Superposition of Deformation Energy in the Linear Elastic Case .....	89
	IV.6.c Deformation Energy in Materials with Linear Elastic Behaviour .....	90
IV.7	Yielding and Rupture Laws .....	92
	IV.7.a General Considerations .....	92
	IV.7.b Yielding Criteria .....	93
	IV.7.b.i Theory of Maximum Normal Stress .....	93
	IV.7.b.ii Theory of Maximum Longitudinal Deformation ..	94
	IV.7.b.iii Theory of Maximum Deformation Energy .....	94
	IV.7.b.iv Theory of Maximum Shearing Stress .....	95
	IV.7.b.v Theory of Maximum Distortion Energy .....	95
	IV.7.b.vi Comparison of Yielding Criteria .....	96
	IV.7.b.vii Conclusions About the Yielding Theories .....	100
	IV.7.c Mohr’s Rupture Theory for Brittle Materials ...	101
IV.8	Concluding Remarks .....	105

IV.9 Examples and Exercises ..... 106

**Part II Strength of Materials**

**V Fundamental Concepts of Strength of Materials** ..... 119

V.1 Introduction ..... 119

V.2 Ductile and Brittle Material Behaviour ..... 121

V.3 Stress and Strain ..... 123

V.4 Work of Deformation. Resilience and Tenacity ..... 125

V.5 High-Strength Steel ..... 127

V.6 Fatigue Failure ..... 128

V.7 Saint-Venant’s Principle ..... 130

V.8 Principle of Superposition ..... 131

V.9 Structural Reliability and Safety ..... 133

    V.9.a Introduction ..... 133

    V.9.b Uncertainties Affecting the Verification  
            of Structural Reliability ..... 133

    V.9.c Probabilistic Approach ..... 134

    V.9.d Semi-Probabilistic Approach ..... 135

    V.9.e Safety Stresses ..... 136

V.10 Slender Members ..... 137

    V.10.a Introduction ..... 137

    V.10.b Definition of Slender Member ..... 138

    V.10.c Conservation of Plane Sections ..... 138

**VI Axially Loaded Members** ..... 141

VI.1 Introduction ..... 141

VI.2 Dimensioning of Members Under Axial Loading ..... 142

VI.3 Axial Deformations ..... 142

VI.4 Statically Indeterminate Structures ..... 143

    VI.4.a Introduction ..... 143

    VI.4.b Computation of Internal Forces ..... 144

    VI.4.c Elasto-Plastic Analysis ..... 145

VI.5 An Introduction to the Prestressing Technique ..... 150

VI.6 Composite Members ..... 153

    VI.6.a Introduction ..... 153

    VI.6.b Position of the Stress Resultant ..... 153

    VI.6.c Stresses and Strains Caused by the Axial Force .. 154

    VI.6.d Effects of Temperature Variations ..... 155

VI.7 Non-Prismatic Members ..... 157

    VI.7.a Introduction ..... 157

    VI.7.b Slender Members with Curved Axis ..... 157

    VI.7.c Slender Members with Variable Cross-Section ... 159

VI.8 Non-Constant Axial Force – Self-Weight ..... 160

VI.9	Stress Concentrations . . . . .	161
VI.10	Examples and Exercises . . . . .	163
<b>VII</b>	<b>Bending Moment . . . . .</b>	<b>189</b>
VII.1	Introduction . . . . .	189
VII.2	General Considerations . . . . .	190
VII.3	Pure Plane Bending . . . . .	193
VII.4	Pure Inclined Bending . . . . .	196
VII.5	Composed Circular Bending . . . . .	200
	VII.5.a The Core of a Cross-Section . . . . .	202
VII.6	Deformation in the Cross-Section Plane . . . . .	204
VII.7	Influence of a Non-Constant Shear Force . . . . .	209
VII.8	Non-Prismatic Members . . . . .	210
	VII.8.a Introduction . . . . .	210
	VII.8.b Slender Members with Variable Cross-Section . . . . .	210
	VII.8.c Slender Members with Curved Axis . . . . .	212
VII.9	Bending of Composite Members . . . . .	213
	VII.9.a Linear Analysis of Symmetrical Reinforced Concrete Cross-Sections . . . . .	216
VII.10	Nonlinear bending . . . . .	219
	VII.10.a Introduction . . . . .	219
	VII.10.b Nonlinear Elastic Bending . . . . .	220
	VII.10.c Bending in Elasto-Plastic Regime . . . . .	221
	VII.10.d Ultimate Bending Strength of Reinforced Concrete Members . . . . .	226
VII.11	Examples and Exercises . . . . .	228
<b>VIII</b>	<b>Shear Force . . . . .</b>	<b>251</b>
VIII.1	General Considerations . . . . .	251
VIII.2	The Longitudinal Shear Force . . . . .	252
VIII.3	Shearing Stresses Caused by the Shear Force . . . . .	258
	VIII.3.a Rectangular Cross-Sections . . . . .	258
	VIII.3.b Symmetrical Cross-Sections . . . . .	259
	VIII.3.c Open Thin-Walled Cross-Sections . . . . .	261
	VIII.3.d Closed Thin-Walled Cross-Sections . . . . .	265
	VIII.3.e Composite Members . . . . .	268
	VIII.3.f Non-Principal Reference Axes . . . . .	269
VIII.4	The Shear Centre . . . . .	270
VIII.5	Non-Prismatic Members . . . . .	273
	VIII.5.a Introduction . . . . .	273
	VIII.5.b Slender Members with Curved Axis . . . . .	273
	VIII.5.c Slender Members with Variable Cross-Section . . . . .	274
VIII.6	Influence of a Non-Constant Shear Force . . . . .	275
VIII.7	Stress State in Slender Members . . . . .	276
VIII.8	Examples and Exercises . . . . .	278

<b>IX</b>	<b>Bending Deflections</b> .....	297
IX.1	Deflections Caused by the Bending Moment .....	297
IX.1.a	Introduction .....	297
IX.1.b	Method of Integration of the Curvature Equation	298
IX.1.c	The Conjugate Beam Method .....	302
IX.1.d	Moment-Area Method .....	304
IX.2	Deflections Caused by the Shear Force .....	308
IX.2.a	Introduction .....	308
IX.2.b	Rectangular Cross-Sections .....	311
IX.2.c	Symmetrical Cross-Sections .....	312
IX.2.d	Thin-Walled Cross-Sections .....	312
IX.3	Statically Indeterminate Frames Under Bending .....	315
IX.3.a	Introduction .....	315
IX.3.b	Equation of Two Moments .....	317
IX.3.c	Equation of Three Moments .....	317
IX.4	Elasto-Plastic Analysis Under Bending .....	320
IX.5	Examples and Exercises .....	323
<b>X</b>	<b>Torsion</b> .....	347
X.1	Introduction .....	347
X.2	Circular Cross-Sections .....	347
X.2.a	Torsion in the Elasto-Plastic Regime .....	353
X.3	Closed Thin-Walled Cross-Sections .....	356
X.3.a	Applicability of the Bredt Formulas .....	361
X.4	General Case .....	362
X.4.a	Introduction .....	362
X.4.b	Hydrodynamical Analogy .....	364
X.4.c	Membrane Analogy .....	365
X.4.d	Rectangular Cross-Sections .....	367
X.4.e	Open Thin-Walled Cross-Sections .....	368
X.5	Optimal Shape of Cross-Sections Under Torsion .....	369
X.6	Examples and Exercises .....	371
<b>XI</b>	<b>Structural Stability</b> .....	389
XI.1	Introduction .....	389
XI.2	Fundamental Concepts .....	391
XI.2.a	Computation of Critical Loads .....	391
XI.2.b	Post-Critical Behaviour .....	393
XI.2.c	Effect of Imperfections .....	396
XI.2.d	Effect of Plastification of Deformable Elements ..	399
XI.3	Instability in the Axial Compression	
	of a Prismatic Bar .....	401
XI.3.a	Introduction .....	401
XI.3.b	Euler's Problem .....	402
XI.3.c	Prismatic Bars with Other Support Conditions ..	403

	XI.3.d	Safety Evaluation of Axially Compressed Members	405
	XI.3.e	Optimal Shape of Axially Compressed Cross-Sections . . . . .	409
XI.4	Instability Under Composed Bending . . . . .		409
	XI.4.a	Introduction and General Considerations . . . . .	409
	XI.4.b	Safety Evaluation . . . . .	414
	XI.4.c	Composed Bending with a Tensile Axial Force . . . . .	416
XI.5	Examples and Exercises . . . . .		416
XI.6	Stability Analysis by the Displacement Method . . . . .		439
	XI.6.a	Introduction . . . . .	439
	XI.6.b	Simple Examples . . . . .	440
	XI.6.c	Framed Structures Under Bending . . . . .	445
		XI.6.c.i Stiffness Matrix of a Compressed Bar . . . . .	445
		XI.6.c.ii Stiffness Matrix of a Tensioned Bar . . . . .	451
		XI.6.c.iii Linearization of the Stiffness Coefficients . . . . .	452
		XI.6.c.iv Examples of Application . . . . .	455
<b>XII</b>	<b>Energy Theorems . . . . .</b>		<b>465</b>
	XII.1	General Considerations . . . . .	465
	XII.2	Elastic Potential Energy in Slender Members . . . . .	466
	XII.3	Theorems for Structures with Linear Elastic Behaviour . . . . .	468
		XII.3.a Clapeyron's Theorem . . . . .	468
		XII.3.b Castigliano's Theorem . . . . .	469
		XII.3.c Menabrea's Theorem or Minimum Energy Theorem . . . . .	473
		XII.3.d Betti's Theorem . . . . .	473
		XII.3.e Maxwell's Theorem . . . . .	477
XII.4	Theorems of Virtual Displacements and Virtual Forces . . . . .		479
		XII.4.a Theorem of Virtual Displacements . . . . .	479
		XII.4.b Theorem of Virtual Forces . . . . .	482
XII.5	Considerations About the Total Potential Energy . . . . .		485
		XII.5.a Definition of Total Potential Energy . . . . .	485
		XII.5.b Principle of Stationarity of the Potential Energy . . . . .	486
		XII.5.c Stability of the Equilibrium . . . . .	486
XII.6	Elementary Analysis of Impact Loads . . . . .		489
XII.7	Examples and Exercises . . . . .		491
XII.8	Chapter VII . . . . .		517
XII.9	Chapter IX . . . . .		518
<b>References . . . . .</b>			<b>523</b>
<b>Index . . . . .</b>			<b>525</b>

**Introduction to the Mechanics of Materials**