

## The Theory of Coherent Radiation by Intense Electron Beams

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# The Theory of Coherent Radiation by Intense Electron Beams

With 42 Figures

 Springer

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## Preface

This book is intended neither as a manual for electrodynamics nor as a monograph dedicated to specific problems of vacuum electronics. It would be naive for the authors to attempt that after numerous brilliant courses of studies, already classical, had been published and after incredibly large number of works, dedicated to research and development of microwave devices, their operation, optimization, etc., had appeared in literature.

One can hardly add anything to the classical theory of the radiation emission by a point charged particle. Almost all the possible configurations of external fields with various boundary conditions for the microwave radiation field had already been investigated. The fundamental effects – Vavilov–Cherenkov radiation, transition radiation in a system with inhomogeneous parameters, and Doppler effect in the case of a relativistic particle moving with acceleration – have been investigated in detail. One can easily find the description of these problems in a large number of specialized monographs and reviews.

At the same time, it is necessary to take a very important logical step to apply the theory to microwaves generation and amplification. The point is that the spectral density of the radiation emitted by a single particle is very low. Multiplication of this density value even by a huge number of individual emitters yields the result of practical interest only in the case of very short waves (e.g., synchrotron radiation) because of the absence of alternative methods. In this situation, coherence of emitters plays the decisive role. It leads to a sharp increase in the spectral–angular brightness of radiation (to be more precise, the field mode composition is implied). Respectively, the efficiency also increases. Surely, coherence, imposed by any of initial conditions, finally has to vanish – at least because of an increase in entropy during the irreversible process of radiation emission. However, in systems, essentially disequilibrium from the viewpoint of thermodynamics, a stage of self-organization can precede self-destruction of the coherence. At this preliminary stage, the coherence is being maintained – or even heightened – because of the radiation reaction (i.e., due to the radiation field backward influence on the motion of particles). As regards low-frequency systems, smaller than the wavelength, the problem

of the coherence maintenance does not arise. However, this problem becomes urgent if one deals with the TWT-type microwave systems with distributed wave-particle interaction.

As a rule, classical electrodynamics of the point particle may be described according to one of the two patterns: it is either “a charged particle in the given field” or “the radiation emission by the charged particle when its motion is prescribed.” One can get the total physical picture by combining these schemes, which is more or less artificial. Applied to a single charged particle, this approach is justified because of the weakness of the radiation reaction. It causes only inessential changes in the particle motion parameters during a time interval in question. However, this method does not fit strong coherent proper fields. There were attempts to elaborate a “self-consistent” electrodynamics of point charged particles. Unfortunately, even the most promising one remained unfinished (P. Dirac, 1940s). In particular, there does not still exist any Lagrangian description of mutual influence of several relativistic particles radiation field taken into account. Therefore, we believe that it would be worthwhile, even if qualitatively, to extend the notions concerning the radiation reaction on a single charge to the case of an ensemble of interacting particles.

Similar difficulties in many particles theories (e.g., the plasma theory) are successfully overcome in the self-consistent field approximation, when the totality of particles is regarded as a charged medium (either hydrodynamic or kinetic continuum). Appearance of the microwave field (i.e., the radiation emission) is regarded then as collective instability of internal degrees of freedom or self-excitation of negative energy proper waves. However, the drawback is that within this approach the spontaneous radiation emission is not taken into account. Besides, in the plasma theory, little attention is paid to relativistic effects and, generally speaking, to the problem of generating microwave fields with prescribed characteristics.

As regards investigations in vacuum electronics, they are aimed, by definition, to optimization of a particular device. Naturally, one is principally interested then in relative advantages of the given construction, while the general physical picture is not being discussed in detail. Surely, there exist many excellent monographs in this field, where physics of the process is profoundly discussed. Notwithstanding this fact, these works, on our opinion, still use a rather specific theory for various devices. We really believe that a common approach, for example, to investigations of Vavilov-Cerenkov radiation, principles of operation of TWT, and Landau damping in collisionless plasmas is not just an attempt to find effective physical parallels but can also be of scientific value. Probably, the most convincing evidence of that is the up-to-date concept of stimulated radiation emission. It combines not only quantum theory of the black body equilibrium and quantum lasers but also purely classical devices of vacuum electronics with distributed interaction.

In more practical sense, this book was stimulated by the quick development of high-current relativistic electronics. By itself, this field is a natural

continuation of traditional vacuum electronics – it just so happened that demand for higher powers stimulated the use of higher currents and higher energies of electron beams. Besides, the advance to shorter wave ranges has conditioned giving traditional slow-wave structures up and using Doppler relativistic effects. All these factors have caused changes in many concepts. For instance, the phenomena, previously treated as unpleasant space charge effects, at present make sometimes the basis of the device operation; the substitution of strictly specified beam quality accelerators for traditional electron guns has cardinally changed the device geometry. In addition, there has arisen a necessity of using open optical cavities or leaving the radiation free at all, etc. Beside, principally new results have been achieved – i.e., the development of the high-current beam technology has enabled advancing into the gigawatt power range, while elaboration of devices working on the basis of Doppler deep transformation (the so-called free electron lasers or FELs) has provided the possibility of stimulating the monochromatic tunable radiation emission even in the soft x-ray band. Because of all of these factors, a large number of specialists in various branches gathered together within this field. All of them had specific concepts, their own experimental and theoretical approaches, different terminology and even their own prejudices. It was our impression that the first discussions somewhat reminded the construction of the Tower of Babel. Surely, later on a mutual understanding was somehow achieved but it is still to be formulated. Periodic literature is of a little help in this aspect.

These factors have determined both the book's composition and the selection of material. In its essence, the book is divided into three parts. The first one is dedicated to the radiation emission by a single relativistic particle. When dealing with the problem of the controllable generation of narrow-band high-power microwaves, the authors have not considered the effects such as wide-band bremsstrahlung which is typical, for instance, of x-ray tubes, while focusing their attention on the prolonged interaction of relativistic particles with a copropagating wave. Based on simple and clear reasoning, this approach enables getting an important piece of information about the field spectral-angular distribution in free space and about the mode composition in an electrodynamic structure. In particular, avoiding Maxwell equations, one can trace the common nature of Cherenkov radiation emitted in media and in slow-wave electrodynamic structures – such as periodic waveguides or diffraction lattices. Similar prolonged interaction might be achieved when a particle is moving along a helix in longitudinal magnetic field or passing through the undulator – a system where the transverse magnetic field alternates in space. In these cases, Doppler normal and anomalous effect plays an essential role because it determines the beam optical activity in the short-wave range, even if the particle is passing through macroscopic structures.

By the way, the synchrotron radiation emission fits the same scheme due to the deep Doppler effect – notwithstanding the fact that for the cyclic motion the wave accompanies the particle only within a short section of the curvilinear

ear trajectory. This fact is rather important because it reveals the common character of the synchrotron and undulator radiation emission, used in FELs.

Presentation of the short-wave undulator radiation emission as a result of the wave scattering by a moving charged particle and prospects of the coherent backward scattering by an intense beam have required the preliminary dwelling on the theory of scattering by a charged particle in the magnetic field – all the more so this problem can be analytically solved under rather loose conditions.

In concluding this section, we present a problem, classical in electrodynamics of the point charged particle – the radiation reaction in relativistic and nonrelativistic cases, also dwelling on the corresponding well-known paradoxes. Paying some attention to the radiation reaction influence on the prolonged particle dynamics, we bear in mind mainly a sequent application of this concept to the case of coherent radiation reaction in a many-particles system.

In fact, similar reasons have dictated our selection of all material for this part. For instance, here the reader can find the total field expansion in potential and solenoidal modes of an arbitrary structure. At the same time, we have left aside the traditional expansion in multipoles because, from the viewpoint of physics, it hardly has any meaning in distributed microwave systems.

The second part – the radiation emission by an ensemble of charged particles – could be regarded as the keystone one. At its beginning, we have presented certain general notions concerning partial coherence of the radiation emitted or scattered in regular structures of various dimensions. Furthermore, we have attempted to describe the stimulated radiation emission as a process of the emitting system self-organization. At the dawn of quantum mechanics development, the “stimulated radiation emission” had been defined as the process reverse to radiation absorption. Later on, the notion of stimulated emission, applied to classical systems with linear spontaneous spectrum, has been regarded as autophasing of individual emitters under the influence of their proper radiation field. It is worth mentioning that both approaches yield completely identical correlation between the spontaneous radiation spectrum and wave amplification under conditions of inverted population. However, the classical approach, which implies mutual autophasing of the particles, is much more illustrative and corresponds better to physics of the process.

Besides, in the second part we have also traced the correlation between discrete and continuous models of the beam. The latter permits applying such an effective tool as the hydrodynamic and kinetic self-consistent equations. It also justifies the use of the concept of negative-energy proper waves and their interaction with electromagnetic waves of the “cold” system. These aspects have been minutely described by an example of the typical problem of an electron beam propagating along a waveguide in a longitudinal magnetic field.

Finally, the third part deals with applying the general ideas to specific schemes. We have presented there beam–plasma systems, gyrotron, and FEL (in spite of our desire, the FEL chapter turned out to be rather bulky, which



is conditioned by the novelty and unusual nature of the device). In accordance with the reasons given above, we tried to avoid coming into details and specificities of the devices schemes. An exception has been made only in cases of absolute necessity – e.g., when we had to explain briefly the principle of operation of the open cavity because of dwelling on the diffraction effects.

The book is written by physicists, for physicists, and about physics. To understand the mathematics involved, one has to handle Fourier and Laplace transforms. The general theory of functions of complex variables is also necessary (within the framework of the university course). The authors tried to describe the models, which can be described analytically, as strictly as it was possible. We do believe that even a limited analytical model is more illustrative than just the results of numerical simulations. Of course, the appropriateness of the model choice is another thing. For those who are ready to take the calculations for granted, “hand-waving” arguments could be sufficient. We tried to use them as often as possible – even taking the risk to sound simplistic.

As regards the references, we can just give our apologies. It is evident that neither the authors nor the reader can physically make acquaintance – even cursory – with all works on the subject. There were even poorer chances to arrange the list of references according to priority – if the latter can be established at all. Therefore, the authors have referred only to the most known manuals and reviews (within the limits of the possible) available both in English and in Russian. We hope that the reader will find useful information in this literature. Original papers from journals are mentioned only in cases of absolute necessity, without giving any priority to them. An excuse, somewhat poor, is that no exception has been made for the authors’ own works.

The book deals with the problems that were being discussed by the authors with many of their colleagues during decades. Thus, it is only fair to consider that these people have also contributed to the concept presented. We are sincerely grateful to all of them, but, unfortunately, it is almost impossible to mention all the names here. Besides, it would be tactless to make the people who have helped us responsible – if even partially – for the authors’ possible omissions or errors. Instead, we would like to pay our greatest respect to those whom we consider our teachers: V. L. Ginzburg, Ya. B. Fainberg, A. V. Gaponov-Grekhov, A. A. Kolomensky, A. I. Akhiezer. . . . We also must mention names of our colleagues: B. Bolotovskiy, Ph. Sprangle, A. Rukhadze, A. Sessler, M. Petelin, J. Nation, N. Ginzburg, V. Bratman, A. Agafonov, and many others. We owe a great deal to them for the scientific exchange and their friendship. The original idea of this book belonged to our late friend V. I. Kurilko, and we dedicate it to his memory. E. Bulyak and I. Bogatyreva have rendered an invaluable contribution to the technical work with the text.

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Kharkov, Moscow  
February 2006

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