Texts and Monographs in Physics

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Relativistic Quantum Mechanics

Second Edition
With 21 Figures

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Preface to the Second Edition

This edition includes five new sections and a third appendix. Most other sections are expanded, in particular Sects. 5.2 and 5.6 on hyperfine interactions.

Section 3.8 offers an introduction to the important field of relativistic quantum chemistry. In Sect. 5.7, the coupling of the anomalous magnetic moment is needed for a relativistic treatment of the proton in hydrogen. It generalizes a remarkable feature of leptonium, namely the non-hermiticity of magnetic hyperfine interactions. In Appendix C, the explicit calculation of the expectation value of an operator which is frequently approximated by a delta-function confirms that the singularity of relativistic wave functions at the origin is correct.

The other three new sections cover dominantly nonrelaticistic topics, in particular the quark model. The coupling of three electron spins (Sect. 3.9) provides also the basis for the three quark spins of baryons (Sect. 5.9). For less than four particles, direct symmetry arguments are simpler than the representions of the permutation group which are normally used in the literature.

Another new topic of this edition is the confirmation of the E^2-dependence of atomic equations by the relativistic energy conservation in radiative atomic transitions, according to the time-dependent perturbation theory of Sect. 5.4. In the quark model, the E^2-theorem applies not only to mesons, but also to baryons as three-quark bound states. Unfortunately, the non-existence of free quarks prevents a precise formulation of the phenomenological "constituent quark model", which remains the most challenging problem of relativistic quantum mechanics.

Karlsruhe, May 2005 *Hartmut M. Pilkuhn*

Preface

Whereas nonrelativistic quantum mechanics is sufficient for any understanding of atomic and molecular spectra, relativistic quantum mechanics explains the finer details. Consequently, textbooks on quantum mechanics expand mainly on the nonrelativistic formalism. Only the Dirac equation for the hydrogen atom is normally included. The relativistic quantum mechanics of one- and two-electron atoms is covered by Bethe and Salpeter (1957), Mizushima (1970) and others. Books with emphasis on atomic and molecular applications discuss also effective "first-order relativistic" operators such as spin-orbit coupling, tensor force and hyperfine operators (Weissbluth 1978). The practical importance of these topics has led to specialized books, for example that of Richards, Trivedi and Cooper (1981) on spin-orbit coupling in molecules, or that of Das (1987) on the relativistic quantum mechanics of electrons. The further development in this direction is mainly the merit of quantum chemists, normally on the basis of the multi-electron Dirac-Breit equation. The topic is covered in reviews (Lawley 1987, Wilson et al. 1991); an excellent monograph by Strange (1998) includes solid-state theory.

Relativistic quantum mechanics is an application of quantum field theory to systems with a given number of massive particles. This is not easy, since the basic field equations (Klein-Gordon and Dirac) contain creation and annihilation operators that can produce unphysical negative-energy solutions in the derived single-particle equations. However, one has learned how to handle these states, even in atoms with two or more electrons. The methods are not particularly elegant; residual problems will be mentioned at the end of Chap. 3. But even there, the precision of these methods is impressive. For example, the influence of virtual electron-positron pairs is included by vacuum polarization, in the form of the Uehling, Kroll-Wichman and Källen-Sabry potentials (Sect. 5.3). For two-body problems, improved methods allow for a fantastic precision, which provides by far the most accurate test of quantum electrodynamics itself.

The present book introduces quantum mechanics in analogy with the Maxwell equations rather than classical mechanics; it emphasizes Lorentz invariance and treats the nonrelativistic version as an approximation. The important quantum field is the photon field, i.e. the electromagnetic field in the Coulomb gauge, but fields for massive particles are also needed. On the other hand, the presentation is very different from that of books on quantum field theory, which include preparatory chapters on classical fields and relativistic quantum mechanics (for example Gross 1993, Yndurain 1996).

The Coulomb gauge is mandatory not only for atomic spectra, but also for the related “quark model” calculations of baryon spectra, which form an important part of the theory of strong interactions. A by-product of an entirely relativistic bound state formalism is a twofold degenerate spectrum, due to explicit charge conjugation invariance. Quark model calculations might benefit from such relatively simple improvements, even when the spectra may eventually be calculated “on the lattice”.

A new topic of this book is a rather broad formalism for relativistic two-body (“binary”) atoms: Nonrelativistically, the Schrödinger equation for an isolated binary can be reduced to an equivalent one-body equation, in which the electron mass is replaced by the “reduced mass”. The extension of this treatment to two relativistic particles will be explained in Chap. 4. The case of two spinless particles was solved already in 1970, see the introduction to Sect. 4.5. The much more important “leptonium” case is treated in Sects. 4.6 and 4.7.

Stimulated by the enormous success of the single-particle Dirac equation, Bethe and Salpeter (1951) constructed a sixteen-component equation for two-fermion binaries. However, increasingly precise calculations disclosed weak points. An effective Dirac equation with a reduced mass cannot be derived from a sixteen-component equation except by an approximate “quasidistance” transformation. On the other hand, such a Dirac equation does follow very directly in an eight-component formalism, in which the relevant S-matrix is prepared as an 8×8-matrix. The principle will be explained in Sect. 4.6, the interaction is added in Sect. 4.7. Like in the Schrödinger equation with reduced mass, the coupling to the photon vector potential operator is treated perturbatively. The famous “Lamb shift” calculation will be presented in Sect. 5.5, extended to the two-body case.

A remarkable property of the new binary equations is the absence of “retardation”. Its disappearance will be demonstrated in Sect. 4.9. Most fermions have an inner structure which requires extra operators already in the single-particle equation. As an example, the fine structure of antiprotonic atoms will be discussed in Sect. 5.6. The Uehling potential is also detailed for these and other “exotic” atoms.

Preparatory studies for this book have been supported by the Volkswagenstiftung. The book would have been impossible without the efforts of my students and collaborators, B. Melić and R. Häckl, M. Malvetti and V. Hund. A textbook by Hund, Malvetti and myself (1997) has provided some of its material.

I dedicate this book to the memory of Oskar Klein.

Karlsruhe, March 2002 *Hartmut M. Pilkuhn*

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