

Scattering and Structures

Bogdan Povh Mitja Rosina

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Essentials and Analogies
in Quantum Physics

Translated by Martin Lavelle

 Springer

Professor Dr. Bogdan Povh
Max-Planck-Institut für Kernphysik
Postfach 10 39 80
69117 Heidelberg
Germany
e-mail: b.povh@mpi-hd.mpg.de

Professor Dr. Mitja Rosina
University of Ljubljana
Department of Physics
Jadranska 19, POB 2964
1000 Ljubljana
Slovenia
e-mail: mitja.rosina@ijs.si

Translator:

Dr. Martin Lavelle
Institut de Física d'Altes Energies
Facultat de Ciències
Universitat Autònoma de Barcelona
08193 Bellaterra (Barcelona)
Spain

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Preface

The initial aim of the book “*Scattering and Structures*”, was to provide a revision course for German students preparing for their oral diploma and Ph.D. examinations where the student is supposed to demonstrate her or his understanding of quantum phenomena by explaining the essential physics without the ballast of the tedious details. The German edition has also been successfully used in students’ seminars and in parallel with standard textbooks.

The attempt to reduce the description and explanation of complicated phenomena to the underlying ideas and formulae has motivated us to extend the framework of the book to many phenomena that seemed suited to such simplification. We hope that the book in its present format can be of interest to students and lecturers as well as to research physicists.

We have much appreciated the discussions with Bernhard Schwingerheuer (Heidelberg) on the new paragraphs of the present edition and Marcus Schwoerer (Bayreuth) for his critique of our original text on the magnetic properties of atoms and on the dispersion in crystals.

We would like to thank Martin Lavelle (Plymouth) for his excellent translation of the book and Jürgen Sawinski (Heidelberg) for his professional work in formatting it.

Preface to the German Edition

La simplicité affectée est une imposture délicate.
La Rochefoucauld

The goal of this book may best be characterised in the words of Ernest Rutherford: “*if you can't explain a result in simple, nontechnical terms, then you don't really understand it*”. In this book “*simple, nontechnical terms*” means language that every physicist can use.

Physics may appear complicated when details cause one to lose sight of the overarching connections. Physics becomes simple when, by the application of a few basic concepts, it is possible to clarify a principle and estimate orders of magnitude. In the following, we will reveal the properties of quantum systems (elementary particles, nucleons, atoms, molecules, quantum gases, quantum liquids and stars) with the help of elementary concepts and analogies between these systems. The choice of topics corresponds to the list of themes that one of the authors (B.P.) used in Heidelberg for the oral physics diploma examination. The book is intended for preparation for the oral diploma examination and for the contemporary Ph.D. defence. Some of the chapters (e.g., 6 und 8) are, though, taken far beyond these examination levels, to make the book of interest to a wider circle of physicists. In a few cases, when we thought that current textbooks do not clearly present the latest developments in physics (e.g., Chap. 3), we have extended the size of the chapter beyond the limit that we have otherwise set ourselves.

In contrast to standard textbooks, no precise derivations are presented. Rather, we have attempted to illuminate physical connections via elementary principles (the uncertainty relation, the Pauli principle), fundamental constants (particle masses, coupling constants) and simple *on the back of an envelope* estimates. One of our models for writing the book in this style

was Victor Weisskopf's lectures for summer students at CERN and his short essays "*Search for Simplicity*" published in the American Journal of Physics in 1985. The individual chapters are constructed as independent units. When we refer to other chapters, this is only to underline the the analogies between different physical systems.

For each chapter, we list textbooks where the general concepts that we use and the simple formulae, which we have not derived, are to be found. All other necessary references are denoted in the text by the authors' names and are also listed at the end of each chapter.

In Chaps. 1–3 and 9, we present scattering as a method for the analysis of quantum systems. In Chaps. 4–6, we consider the construction of the simplest composites of the electromagnetic and strong interactions: atoms and hadrons. The interatomic forces that lead to the construction of molecules are treated in Chaps. 7 and 8, while the analogous force in the strong interaction, the nuclear force, is briefly discussed in Chap. 10. Degenerate systems of fermions and bosons, from quantum gases through to neutron stars, are the main theme of Chaps. 11–15. In Chap. 16, we mention some of the open questions of contemporary elementary particle physics.

It is obvious that errors can creep into any attempt to represent complex phenomena elegantly with the help of "physical intuition". We ask critical readers to point out any such slips to us. We would be happy to receive ideas for how further examples of quantum phenomena can be grasped and made plausible *on the back of an envelope*. Suggestions for how overly lengthy discussions could be shortened without any loss of clarity are also very welcome.

Special thanks for proposals improving the content, style and language of the whole book are due to Christoph Scholz (Reilingen) and Michael Treichel (Munich). The current title of the book was also suggested to us by Michael Treichel.

We received valuable criticism on the first two chapters from Paul Kienle (Munich) and on the nuclear physics chapters from Peter Brix (Heidelberg). We have discussed the treatment of chiral symmetry breaking at length with Jörg Hüfner (Heidelberg) and Thomas Walcher (Mainz). We received private tuition

in phase transitions and solid state physics from Franz Wegner (Heidelberg) and Reimer Kühn (Heidelberg). Samo Fišinger (Heidelberg) helped us to formulate the section on proteins. The chapters on quantum gases and quantum liquids were produced with the help of Allard Mosk (Utrecht) and Mattias Weidemüller (Heidelberg). Claus Rolfs (Bochum) thoroughly corrected the chapter on stars. We discussed in detail the newest results from neutrino research with Stephan Schönert (Heidelberg). Ingmar Köser and Claudia Ries have taken great pains to translate the manuscript of the book into good German. Jürgen Sawinski was responsible for the layout and producing the figures.

Working with Wolf Beiglböck and Gertrud Dimler of Springer was, as ever, a pleasure.

Heidelberg,
July 2002

Bogdan Povh
Mitja Rosina

Prelude

The most powerful emperor of the 13th dynasty had led the Middle Kingdom to new glory. A new picture, that of a dragon, the symbol of the power of the empire, was planned to decorate his palace. He commissioned the best artist of the empire with the task of producing the picture.

After 2 years, the artist eventually appeared with his picture before the emperor. As he unrolled the canvas, the emperor glimpsed a green background with a yellow, slightly snaking line on it.

“You needed 2 years for this?” demanded the emperor in a rage. Convinced that the artist was mocking him, he let him be taken away and condemned him to death. A wise advisor of the emperor said, however: “let us, oh great emperor, personally see what the artist has done for the last 2 years.” When the emperor and his advisor entered the studio of the artist, they saw over 700 pictures lined up according to the order of their production. The artist had painted a new picture each day. The first pictures showed the dragon in all possible detail. The later ones lacked more and more of the insignificant details, but the essence of the dragon was ever clearer. The last pictures were already very similar to that which the artist had brought to him. “Now I see”, said the emperor, “the essence of the dragon has been perfectly represented by the artist.”

The emperor pardoned the artist.

Chinese fairy tale

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