

Lecture Notes in Statistics

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Estimation in Conditionally Heteroscedastic Time Series Models

 Springer

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Library of Congress Control Number: 2004115047

ISBN 3-540-21135-7 Springer Berlin Heidelberg New York

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Cover design: *design & production*, Heidelberg
Typesetting by the author using L^AT_EX
Printed on acid-free paper

40/3142YL-5 4 3 2 1 0

To my parents

Preface

This monograph arose from my PhD thesis, which was written under the supervision of Prof. T. Mikosch at the universities of Groningen and Copenhagen. The extensions and generalizations of the PhD material were carried out at RiskLab, Department of Mathematics, ETH Zurich.

Acknowledgements

I am indebted to my PhD supervisor Prof. T. Mikosch who supported me throughout my PhD time. Without his inputs and continuous criticism this monograph would not exist. I am also grateful for the advice of numerous other people who were kind enough to talk with me about aspects related to this work.

In particular I would like to thank all those who received my work in a positive way. The list starts with the PhD committee: Prof. M. Sørensen (Copenhagen), Prof. F. Liese (Rostock) and Prof. A. McNeil (Zurich). During my year at RiskLab I was invited by Prof. M. Deistler (Vienna), Profs. J.-P. Kreiss and M. Neumann (Braunschweig), Prof. R. Dahlhaus and Dr. S. Subba Rao Tata (Heidelberg) and Prof. U. Schmock (Vienna). On this occasion I would like to express my gratitude for all these invitations and for the extraordinary hospitality that I experienced. The discussions I had were a great source of motivation. In a workshop I had the opportunity to meet “Berkes et al.” (i.e. Profs. I. Berkes, L. Horváth and P. Kokoszka). Their paper [8] had inspired me to consider likelihood inference in conditionally heteroscedastic time series models. Furthermore I had email contact with Profs. F. Liese (Rostock) and A. Mokkadem (Paris). Thank you very much for answering all my questions! I also received positive feedback from Profs. G. Balkema, F. Drost and Q. Yao.

I would like to acknowledge the generous support of RiskLab Zurich without which I could not have improved my PhD thesis. My stay at RiskLab was organized by Profs. P. Embrechts and A. McNeil. Special thanks go to Dr. M. Studer. He read my monograph very carefully and made many suggestions for improvements.

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To my parents I owe moral support, and for this reason I dedicate this booklet to them.

Zurich,
September 2004

Daniel Straumann

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