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Integral Closure

Rees Algebras, Multiplicities, Algorithms

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This is for Aurea

Preface

There are many ways of looking at the properties of an ideal I of a Noetherian ring R, an R–module E, or even an R–algebra B and the geometric objects they represent. We single out the following aspects:

- Syzygies & Hilbert functions
- Polynomial relations
- Linkage & deformation
- Integral closure
- Primary decomposition
- Complexity of computations

These aspects often arise as outcomes of processes required to understand the deep structure of the subvariety defined by I, but also in many constructions defined on I (and similarly on E or B.) One way to look at them all is via the filtrations that initialize on I, giving rise to Rees algebras and Hilbert functions. The examination of these algebras and functions embraces all of these issues and some more.

Blowup algebras realize as rings of functions the process of blowing up a variety along a subvariety. This book will focus on Rees rings of ideals (and their generalizations), the most ubiquitous of those algebras. It is primarily aimed however at providing a survey of several recent developments (and some not so recent) on integral closure in many guises. It will be realized by looking at the numerical invariants, special divisors and attached algebras whose interplay assists in understanding the arithmetic of various algebras. The underlying techniques are fundamentally computations of cohomology on suitable algebras. It seeks out those regularity properties usually associated with Cohen–Macaulayness and normality. The first controls many of the numerical invariants of the blowup, while the latter is a required ingredient of desingularization.

The required extension of these problems, from ideals to modules, is very apparent already in the comparison of associated graded rings with Rees algebras of conormal modules. Given the centrality of finding integral closures of algebras, our subject finds its full theme: the study of integral closure of algebras, ideals and modules.

Emphasis will be placed on determining these invariants and properties from a description of the ring, ideal or module by generators and relations. Open problems and basic techniques will be stressed at the expense of individual results. Another rule of navigation used was that given the choice between two approaches to a topic, the path that seemed more constructive to the author was taken—unless the sheer beauty of the more abstract method made the choice inevitable.

A limited effort in this direction by the author [Va94b] was slightly premature, as the subject was experiencing just then an explosion of activity. Given the rate of change in the subject, these notes will also be premature, with probability of one, whenever they appear. It is worth emphasizing that while this book and [Va94b] deal with some of the same algebras, the focus here is on normalization while the other book was mainly aimed at the study of the Cohen–Macaulay property. However, these topics have grown so much that the subject deserves an exclusive treatment. We shall nevertheless touch on several of these activities.

The reader will observe an imbalance of attention given to certain topics. They are often an expression of ignorance/lack of expertise by the author, or simply that the topic is undergoing so many developments that it would be premature to stop & look around.

We are very grateful to many colleagues who over the years have shared directly (or indirectly via their publications) their ideas on these subjects with the author. Others had a very direct involvement as co-authors in some of the joint research reported here or freely took on the task of proofreading parts of the manuscript. To all of them, but particularly to Joe Brennan, Alberto Corso, Sam Huckaba, Jooyoun Hong, Claudia Polini, Aron Simis, Amelia Taylor, Bernd Ulrich and Rafael Villarreal, the author gives his heartfelt thanks. The support and care of Martin Peters and Ruth Allewelt at Springer is much appreciated. Finally, we are grateful to James Wiegold for his careful editorial work on the manuscript.

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