Dynamics Reported

Volume 1

Dynamics Reported

The rapid development in the area of Dynamical Systems continually calls for comprehensive presentations of the current topics, in particular because Dynamical Systems are closely related to many other areas of mathematics and are of utmost interest to engineers, scientists, etc.

Dynamics Reported is a book series. Each volume contains about four or five longer articles of up to 60 pages. Each article treats an important subject of current research on a high scientific *and* didactic level. The main results are accompanied by carefully written proofs to ensure an adequate degree of completeness of each article. *Dynamics Reported* is published bi-annually or annually.

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Volume 1

U. Kirchgraber and H. O. Walther
Managing Editors

Mather Sets for Twist Maps and Geodesics on Tori
V. Bangert

Connecting Orbits in Scalar Reaction Diffusion Equations
P. Brunovský and B. Fiedler

Qualitative Theory of Nonlinear Resonance by Averaging and Dynamical Systems Methods
James Murdock

An Algorithmic Approach for Solving Singularly Perturbed Initial Value Problems
K. Nipp

Exponential Dichotomies, the Shadowing Lemma and Transversal Homoclinic Points
Kenneth J. Palmer

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Preface

Dynamics Reported reports on recent developments in dynamical systems theory.

Dynamical systems theory of course originated from ordinary differential equations. Today, dynamical systems theory covers a much larger area, including dynamical processes described by functional and integral equations, by partial and stochastic differential equations, etc. Dynamical systems theory has evolved remarkably rapidly in the recent years. A wealth of new phenomena, new ideas and new techniques proved to be of considerable interest to scientists in rather different fields. It is not surprising that thousands of publications on the theory itself and on its various applications have appeared and still will appear.

Dynamics Reported presents carefully written articles on major subjects in dynamical systems and their applications, addressed not only to specialists but also to a broader range of readers. Topics are advanced while detailed exposition of ideas, restriction to *typical* results, rather than to the *most general* ones, and last but not least lucid proofs help to gain an utmost degree of clarity.

It is hoped that *Dynamics Reported* will stimulate exchange of ideas among those working in dynamical systems and moreover will be useful for those entering the field.

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