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Armin Hemmerling

## Labyrinth Problems

Labyrinth-Searching Abilities of Automata

This book deals with the main results and open problems of labyrinth theory which has to be considered as a branch of complexity theory within theoretical computer science. The central problems treated here concern the searching of certain embedded graphs by several kinds of automata able to move in these graphs.

Dieser Teubner-Text behandelt die wichtigsten Resultate und offenen Probleme der Labyrinth-Theorie, die als Zweig der Kompliziertheorie innerhalb der theoretischen Informatik anzusehen ist. Im Zentrum der Untersuchungen stehen hier Probleme des Absuchens gewisser eingebetteter Graphen durch verschiedene Typen von Automaten, die in der Lage sind, sich auf diesen Graphen zu bewegen.

Ce livre est consacré au traitement des résultats les plus importants de la théorie des labyrinthes et à celui des problèmes qui restent ouverts. Cette théorie est ici considérée une branche de la théorie de la complexité en informatique théorique. Au centre de ces recherches se trouve le problème de l'explorations de certains graphes immergés par différents types d'automates qui sont en mesure de se mouvoir sur ces graphes.

Книга содержит главные результаты, а также нерешённые проблемы теории лабиринтов, которая рассматривается как ветвь теории сложности в области теоретической информатики. При этом, в центре исследований стоят проблемы обхода некоторых погруженных графов, посредством различных видов автоматов, обладающих способностью двигаться по этим графам.

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