

Gauge Theories

of the Strong and Electroweak Interaction

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Preface

The gauge theories for the strong and electroweak interaction have become the Standard Model of particle physics. They realize in a consistent way the requirements of quantum theory, special relativity, and symmetry principles. For the first time, we have a consistent theory of the fundamental interactions that allows for precision calculations for many experiments. The Standard Model has, up to now, successfully passed all experimental tests. This success establishes the importance of gauge theories, despite the fact that gravity is not included and that the Standard Model is most likely an effective theory resulting from the low-energy limit of a more fundamental theory.

The aim of this book is to present the basic ideas and concepts, the technical tools, and the predictions of the gauge theories for the fundamental constituents of matter and their interactions: Quantum Chromodynamics for the strong interaction and the Electroweak Standard Model for the unified electromagnetic and weak interaction.

The first edition of this book, P. Becher, M. Böhm and H. Joos, *Eichtheorien der starken und elektroschwachen Wechselwirkung*, appeared in German in 1981. At that time, gauge theories were not yet really supported by experiment. The W and Z bosons were not discovered, the existence of the gluon was not confirmed, and the top-quark had not been found. Presently, after a lot of theoretical and experimental work, gauge theories are much better understood, many precision calculations exist, and a huge amount of experimental data is available. All this has put the gauge theories on a firm ground and required a thorough revision of the original book. This third edition is almost completely rewritten and largely extended. We tried to cover the most important developments and to guide the reader to the modern applications of gauge theories in elementary particle physics.

As before, this book is written for students who have passed the standard course in theoretical physics and an introductory course in particle physics. The book is also addressed to experimental and theoretical physicists who want to become familiar with the established treatment of the fundamental

interactions. We tried to cover a large part of the applications of gauge theories, to present the material in a self-contained form, and to show in detailed calculations, as far as possible, how the results are obtained. In order to help the reader to get along with this huge amount of material we supply a detailed index and a large list of references. Evidently, the list of references cannot be exhaustive. We tried to focus on original papers, review papers, and articles that, in our opinion, should help the reader in understanding the subjects.

In the first part of the book, we review the basic concepts of the phenomenology of particle physics and relativistic quantum field theory. Then, we treat the quantum theory of gauge fields in detail. We work out the invariant perturbation theory including Feynman-diagram techniques and renormalization, which is the basis for many precision calculations. The topological properties of gauge theories are important for their non-perturbative evaluation. We discuss, in particular, instanton solutions of gauge theories and gauge theories on the lattice. This knowledge of the structure of gauge theories and of the methods for their evaluation is then applied to Quantum Chromodynamics and the Electroweak Standard Model. We conclude with short a survey of grand unified theories and supersymmetry.

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Contents

1	Phenomenological basis of gauge theories of strong, electromagnetic, and weak interactions	1
1.1	Elementary particles and their interactions	1
1.1.1	Leptons and quarks as fundamental constituents of matter . .	2
1.1.2	Fundamental interactions	4
1.2	Elements of relativistic quantum field theory	5
1.2.1	Basic concepts of relativistic quantum field theory	6
1.2.2	Lie algebras and Lie groups	18
1.2.3	Conserved currents and charges	24
1.3	The quark model of hadrons	29
1.3.1	Quantum numbers and wave functions of hadrons in the quark model	29
1.3.2	Quark model with colour	35
1.3.3	The concept of quark dynamics—quarkonia	36
1.4	Basics of the electroweak interaction	39
1.4.1	Electroweak interaction of leptons	41
1.4.2	Electroweak interaction of hadrons	47
1.5	The quark–parton model	57
1.5.1	Scaling in deep-inelastic lepton–nucleon scattering	57
1.5.2	The parton model	63
1.5.3	Applications of the simple parton model	68
1.5.4	Universality of the parton model	72
1.6	Higher-order field-theoretical effects in QED	76
1.6.1	QED as a quantum field theory	76

VI Contents

1.6.2	A test of QED: the magnetic moment of the muon	78
1.7	Towards gauge theories of strong and electroweak interactions	81
	References to Chapter 1	82
2	Quantum theory of Yang–Mills fields	85
2.1	Green functions and S -matrix elements	86
2.1.1	The principles of quantum field theory	86
2.1.2	Green functions	88
2.1.3	S -matrix elements and the LSZ formula	95
2.1.4	Connected Green functions and vertex functions	97
2.1.5	Scattering of composite particles	101
2.2	Path-integral representation of quantum field theory	104
2.2.1	Functional calculus	104
2.2.2	Generating functionals of Green functions	111
2.2.3	Functional-integral representation of the S matrix	115
2.2.4	The field-theoretical path integral	116
2.2.5	Feynman rules and path integral	122
2.2.6	Ward identities and equations of motion for Green functions .	127
2.3	Local gauge invariance	133
2.3.1	Local gauge invariance in QED	134
2.3.2	Geometry of non-abelian gauge symmetry	136
2.3.3	Yang–Mills field theories	143
2.4	Path-integral formulation of gauge theories	148
2.4.1	Path-integral quantization of gauge theories	149
2.4.2	Feynman rules for gauge theories	157
2.4.3	BRS invariance and Slavnov–Taylor identities	161
2.4.4	The background-field method	178
2.5	Renormalization of quantum field theories	188
2.5.1	Divergences and renormalization	188
2.5.2	Calculation of one-loop corrections	207
2.5.3	One-loop renormalization of gauge theories	219

2.5.4	Sketch of the proof of renormalizability of gauge theories . . .	231
2.6	Renormalization group	252
2.6.1	Renormalization-group equation	252
2.6.2	Renormalization-group function and anomalous dimensions of massless gauge theories	260
2.6.3	Relation between different renormalization schemes	265
2.6.4	Running unrenormalized coupling constant	267
2.7	Anomalies	268
2.7.1	The triangle-graph anomaly	269
2.7.2	Anomalies in gauge theories	279
2.8	Infrared and collinear singularities	285
2.8.1	The origin of mass singularities	286
2.8.2	Infrared singularities	294
2.8.3	Collinear singularities in QED	305
2.9	Non-perturbative aspects of gauge theories	328
2.9.1	Topological quantum numbers	329
2.9.2	The index theorem	339
2.9.3	Path integral and topology	347
2.10	Lattice approximation of gauge theories	379
2.10.1	Basics of lattice approximation	379
2.10.2	Strong-coupling approximation	385
2.10.3	Numerical methods	390
2.10.4	Transition to the continuum	397
2.10.5	Finite-size effects	404
2.10.6	Lattice approximation of fermionic interactions	406
	References to Chapter 2	415

VIII Contents

3	Quantum Chromodynamics	426
3.1	Asymptotic freedom of QCD	427
3.1.1	The running coupling constant of QCD	427
3.1.2	Importance of higher-order corrections	430
3.1.3	The running quark masses of QCD	432
3.2	QCD in deep-inelastic scattering	435
3.2.1	The field-theoretical approach to the parton model	436
3.2.2	QCD corrections to the parton model	447
3.2.3	Evolution equations	458
3.2.4	Experimental tests of QCD	464
3.3	Perturbative Quantum Chromodynamics	469
3.3.1	One-loop corrections to the parton model	470
3.3.2	Factorization	476
3.3.3	Factorization and the operator-product expansion	479
3.3.4	Lepton-pair production in hadron-hadron scattering	484
3.3.5	Jet cross sections	488
3.3.6	Total hadronic cross section	494
3.4	Heavy-quark effective theory (HQET)	497
3.4.1	The Lagrangian of HQET	498
3.4.2	Symmetries of HQET	502
3.4.3	Applications of HQET	504
3.5	Light quarks and chiral perturbation theory	507
3.5.1	Chiral symmetry of massless QCD	507
3.5.2	Pion-pole dominance and effective low-energy theory	510
3.5.3	The non-linear σ model	512
3.5.4	Breaking of chiral invariance	513
3.5.5	Applications of chiral perturbation theory	516
3.6	Results of lattice approximation of QCD	518
3.6.1	The hadron spectrum	519
3.6.2	Glue balls	524

3.6.3	The connection between long- and short-distance physics: non-perturbative renormalization group	526
3.7	Quark confinement	529
3.7.1	The Wilson criterion	529
3.7.2	Quark confinement in strong-coupling approximation	534
3.7.3	The string picture	536
3.7.4	Long-range correlations of the QCD vacuum	548
3.8	A test of QCD: the running of α_s	554
	References to Chapter 3	557
4	Gauge theories of the electroweak interaction	566
4.1	Spontaneous symmetry breaking	567
4.1.1	Spontaneous breaking of a global symmetry	568
4.1.2	Spontaneous breaking of a gauge symmetry	580
4.2	The Standard Model of the Electroweak Interaction	591
4.2.1	The Lagrangian of the Electroweak Standard Model	592
4.2.2	The Lagrangian in the physical basis	596
4.3	Simple applications of the Electroweak Standard Model	604
4.3.1	W-pair production in e^+e^- annihilation	604
4.3.2	Production and decay of the Higgs boson	613
4.4	Quantization of the Electroweak Standard Model	622
4.4.1	Gauge fixing and Faddeev–Popov fields	622
4.4.2	BRS symmetry and physical fields	625
4.4.3	Slavnov–Taylor identities	628
4.4.4	Lee identities	630
4.4.5	The background-field method for the Electroweak Standard Model	638
4.4.6	Charge universality in the Electroweak Standard Model	642
4.4.7	The Goldstone-boson equivalence theorem	646
4.5	Renormalization of the Electroweak Standard Model	651
4.5.1	The renormalization transformation in the on-shell scheme	652

X Contents

4.5.2	Renormalization conditions	655
4.5.3	Explicit form of renormalization constants	661
4.5.4	Renormalization within the background-field method	666
4.5.5	Mass renormalization for unstable particles	668
4.6	Electroweak radiative corrections	671
4.6.1	Fermionic contributions to the gauge-boson self-energies . . .	672
4.6.2	Parameter relations in higher orders	675
4.6.3	Decay widths of the weak gauge bosons	684
4.6.4	Z-boson physics	689
4.6.5	Precision tests of the electroweak interaction	704
4.6.6	Status of the Electroweak Standard Model and perspectives .	706
	References to Chapter 4	709
5	Extensions of the Standard Model	716
5.1	Grand unified theories (GUTs)	717
5.1.1	Unification of coupling constants	718
5.1.2	Proton decay	721
5.1.3	Hierarchy (fine-tuning) problem	723
5.1.4	SU(5) GUT	723
5.2	Supersymmetry	732
5.2.1	Supersymmetry algebra	732
5.2.2	The chiral multiplet and the Wess–Zumino model	733
5.2.3	Improved ultraviolet properties of supersymmetric theories . .	734
5.2.4	The Minimal Supersymmetric Standard Model (MSSM) . . .	736
5.2.5	Supersymmetric grand unification	738
	References to Chapter 5	740

A	Appendix	742
A.1	Conventions	742
A.1.1	Minkowski space	742
A.1.2	Euclidean space	750
A.1.3	Field theory	751
A.2	Feynman rules for the Electroweak Standard Model	758
	General references	765
	Index	766

Corrections to this book

A list of misprints and corrections to this book is posted on the World-Wide Web at the URL <http://www.hep.psi.ch/denner/GTupdates.html>, or can be obtained by writing to the authors. We would be grateful, if you would report additional errors in the book to Ansgar.Denner@psi.ch.