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John Guaschi · Daniel Juan-Pineda
Silvia Millán López

The Lower Algebraic K-Theory of Virtually Cyclic Subgroups of the Braid Groups of the Sphere and of $\mathbb{Z}[B_4(\mathbb{S}^2)]$

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John Guaschi
Laboratoire de Mathématiques Nicolas Oresme UMR CNRS 6139
Université de Caen Normandie, Normandie Université
Caen, France

Daniel Juan-Pineda
Centro de Ciencias Matemáticas
Universidad Nacional Autónoma de México
Morelia, Michoacán, Mexico

Silvia Millán López
Colegio de Bachilleres del Estado de Tlaxcala
Tlaxcala, Mexico

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Preface

We study K-theoretical aspects of the braid groups $B_n(\mathbb{S}^2)$ on n strings of the 2-sphere, which by results of the second two authors are known to satisfy the Farrell–Jones fibred isomorphism conjecture [1]. In light of this, in order to determine the algebraic K-theory of the group ring $\mathbb{Z}[B_n(\mathbb{S}^2)]$, one should first compute that of its virtually cyclic subgroups, which were classified by D. L. Gonçalves and the first author [2]. We calculate the Whitehead and K_{-1}-groups of the group rings of the finite subgroups (dicyclic and binary polyhedral) of $B_n(\mathbb{S}^2)$ for all $4 \leq n \leq 11$. Some new phenomena occur, such as the appearance of torsion for the K_{-1}-groups. We then go on to study the case $n = 4$ in detail, which is the smallest value of n for which $B_n(\mathbb{S}^2)$ is infinite. We show that $B_4(\mathbb{S}^2)$ is an amalgamated product of two finite groups, from which we are able to determine a universal space for proper actions of the group $B_4(\mathbb{S}^2)$. We also calculate the algebraic K-theory of the infinite virtually cyclic subgroups of $B_4(\mathbb{S}^2)$, including the Nil groups of the quaternion group of order 8. This enables us to determine the lower algebraic K-theory of $\mathbb{Z}[B_4(\mathbb{S}^2)]$.

Caen, France — John Guaschi
Morelia, Mexico — Daniel Juan-Pineda
Tlaxcala, Mexico — Silvia Millán López

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