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Massimiliano Berti • Jean-Marc Delort

Almost Global Solutions of Capillary-Gravity Water Waves Equations on the Circle

Springer

U M I Unione Matematica Italiana

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Preface

The goal of this monograph is to prove that any solution of the Cauchy problem for the capillary-gravity water waves equations, in one space dimension, with periodic, even in space, initial data of small size ϵ, is almost globally defined in time on Sobolev spaces; i.e. it exists on a time interval of length of magnitude ϵ^{-N} for any N, as soon as the initial data are smooth enough, and the gravity-capillary parameters are taken outside an exceptional subset of zero measure. In contrast to the many results known for these equations on the real line, with decaying Cauchy data, one cannot make use of dispersive properties of the linear flow. Instead, our method is based on a normal form procedure, in order to eliminate those contributions to the Sobolev energy that are of lower degree of homogeneity in the solution.

Since the water waves equations are a quasi-linear system, usual normal form approaches would face the well-known problem of losses of derivatives in the unbounded transformations. In this monograph, to overcome such a difficulty, after a paralinearization of the capillary-gravity water waves equations, necessary to obtain energy estimates, and thus local existence of the solutions, we first perform several paradifferential reductions of the equations to obtain a diagonal system with constant coefficients symbols, up to smoothing remainders. Then we may start with a normal form procedure where the small divisors are compensated by the previous paradifferential regularization. The reversible structure of the water waves equations, and the fact that we look for solutions even in x, guarantees a key cancellation which prevents the growth of the Sobolev norms of the solutions.

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