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Andrea Loi • Michela Zedda

Kähler Immersions of Kähler Manifolds into Complex Space Forms

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This book has been awarded the 2017 Book Prize of the Unione Matematica Italiana.

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Preface

The study of Kähler immersions of a given real analytic Kähler manifold into a finite- or infinite-dimensional complex space form originates from the pioneering work of Eugenio Calabi [10]. With a stroke of genius, Calabi defined a powerful tool, a special (local) potential called the *diastasis function*, which allowed him to obtain necessary and sufficient conditions for a neighbourhood of a point to be locally Kähler immersed into a finite- or infinite-dimensional complex space form. As an application of this criterion, he also provided a classification of (finite-dimensional) complex space forms admitting a Kähler immersion into another. However, a complete classification of Kähler manifolds admitting a Kähler immersion into complex space forms is not known, not even when the Kähler manifolds involved are of great interest, e.g. when they are Einstein or homogeneous spaces. In fact, the diastasis function is not always explicitly given, and most of the time Calabi's criterion, although theoretically impeccable, is difficult to apply. Nevertheless, throughout the last 60 years many mathematicians have worked on the subject and many interesting results have been obtained.

The aim of this book is to describe Calabi's original work, to provide a detailed account of what is known today on the subject and to point out some open problems.

Each chapter begins with a brief summary of the topics discussed and ends with a list of exercises to test the reader's understanding.

Apart from the topics discussed in Sect. 3.1 of Chap. 3, which could be skipped without compromising the understanding of the rest of the book, the prerequisites for this book are a basic knowledge of complex and Kähler geometry (treated, e.g. in Moroianu's book [61]).

The authors are grateful to Claudio Arezzo and Fabio Zuddas for their careful reading of the text and for their valuable comments, which have greatly improved the book's exposition.

Cagliari, Italy — Andrea Loi
Parma, Italy — Michela Zedda
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