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Editors

Visualizing Mathematics

The Role of Spatial Reasoning in Mathematical Thought

Springer

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Foreword

In 2016, we were approached by series editor, Dr. Jinfa Cai, with a novel idea—invite authors from the fields of developmental psychology and mathematics education to write about their work on spatial visualization and mathematics, and then ask them to write commentaries on one another's chapters. The goal was to provide a unique view of research on this topic that encompassed both disciplines, as well as foster cross-field communication and intellectual synergy. We eagerly took up the challenge and invited scholars whose work we knew to be at the forefront of our respective fields. The chapters and commentaries contained in this volume are the products of this esteemed group. They reflect the state of the art in research on spatial visualization and mathematics from at least two perspectives. They highlight important new contributions, but they also reveal the fault lines between our respective disciplines. The commentaries insightfully point out some of these fault lines, as well as the immense common ground and the potential for deeper collaboration in the future.

The basic question of how spatial skill relates to mathematics has received steady attention over the years. In psychology, most of this work has focused on long-term outcomes in STEM fields for individuals with more advanced spatial skill (e.g., Wai, Lubinski, & Benbow, 2009), the possibility that spatial deficits contribute to poor mathematics outcomes in children (e.g., Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007), and the use of materials that physically embody (via spatial relations) abstract mathematics concepts (see Mix, 2010, for a review). Running through these disparate research programs is the shared notion that spatial thinking plays a major role in understanding mathematics, but it has not been addressed head on in psychology until recent years.

In mathematics education, Clements and Battista, in their 1992 research review, address just this issue. They wrote that both Hadamard and Einstein (renown mathematicians) claimed that much of the thinking required in higher mathematics is spatial, and they cited positive correlations between spatial ability and mathematics achievement at all grade levels. However, even in that time period, the relations between spatial thinking and learning nongeometric concepts did not seem straightforward, and there were conflicting findings. For some tasks, having high-spatial skill seemed to improve performance, whereas in other tasks, processing mathematical information using verbal-logical reasoning enhanced performance compared to students who processed the information visually. Other mathematics education researchers countered that the understanding of some low-spatial students who did well in mathematics was instrumental, whereas high-spatial students' understanding was more relational, a difference often not captured by classroom or standardized assessments. Clements and Battista concluded that even though there was reason to believe that spatial reasoning is important in students' learning and use of mathematical concepts—including nongeometric concepts—the role that such reasoning plays in this learning remained elusive.

Possibly because of this elusiveness, interest in the topic waned in mathematics education. However, currently there is intense interest in this general topic in both psychology and mathematics education due to its potential educational benefits (Newcombe, 2010) and the insights into the relations found through extensive and detailed student interviews (Bruce et al., 2017; Davis et al., 2015). The chapters contributed to this volume represent various approaches to advancing this work in education or moving the work in both fields toward educational application.

The developmental psychology chapters tended to focus on the underlying mental representations used to understand mathematics, and the extent to which these representations already involve, or could be improved by spatial processing. Cipora, Schroeder, Soltanlou, and Nuerk provide a detailed analysis of the link between spatial and numerical processing purportedly demonstrated by spatial-numerical association (SNA) or mental number line effects. They conclude that spatial skills provide a crucial tool for understanding mathematics, but this relation may not be realized in the form of a fixed mental number line. Congdon, Vasileyva, Mix, and Levine examine a deep psychological structure that may underlie a range of mathematics topics—namely, the structure involved in identifying and enumerating spatial units of measurement. They argue that mastery of this structure has the potential to support mathematics learning throughout the elementary grades and perhaps head off misconceptions related to fractions, proportions, and conventional later on. Similarly, Jirout and Newcombe focus on another spatial relation with strong ties to mathematics—namely, relative magnitude—outline its potential role in improving instruction on whole number ordering, fractions, and proportions. Casey and Fell discuss the difference between general spatial skill and spatial skill instantiated in specific mathematics problems, concluding that the most effective way to leverage spatial training to improve mathematics outcomes is likely the latter. They highlight a number of instructional techniques from existing curricula that successfully use spatial representations. Finally, Young, Levine, and Mix considered the multidimensional nature of spatial processing and mathematics processing and the inherent complexity involved in identifying possible instructional levers. Following a critique of the existing literature, including recent factor analytic approaches, they conclude with a set of recommendations for improving these approaches and applying what is already known in educational settings.

The mathematics education chapters discuss the spatial processes involved in specific topics in mathematics. Sinclair, Moss, Hawes, and Stephenson examine how children can learn "*through* and *from* drawing," focusing on spatial processes and concepts in primary school geometry. They argue that drawing is not innate but can be improved, and they illustrate through fine-grained analysis how the potential benefits of geometric drawing can be realized in classrooms. Gutiérrez, Ramírez, Benedicto, Beltrán-Meneu, and Jaime analyze the spatial reasoning of mathematically gifted secondary school students as they worked on a collaborative, communication-intensive, task in which they were shown orthogonal projections of cube buildings along with related verbal information. The authors related the objectives of students' actions and their visualization processes and students' solution strategies and cognitive demand. Herbst and Boileau argue that high school geometry instruction can do more than provide names for 3D shapes and formulas for finding surface area and volume. They illustrate, and invite reflection on their design of, a 3D geometry modeling activity in which students write and interpret instructions for how to move pieces of furniture up an L staircase. Lowrie and Logan discuss how the frequency of encountering, and interacting with, information in visual/graphic format, including on the web, has increased our need for research on the role of spatial reasoning in students' encoding and decoding of information in mathematics. To this end, they analyze the representational reasoning of students engaged in tasks that permit different types of representations, from diagrams to equations. Battista, Frazee, and Winer describe the spatial processes involved in reasoning about the geometric topics of measurement, shapes, and isometries. They introduce, and use in their analysis, the construct of *spatial-numerical linked structuring* as the coordinated process in which numerical operations on measurement numbers are linked to spatial structuring of, and operation on, the measured objects in a way that is consistent with properties of numbers and measurement.

As the chapters and commentaries illustrate, there are still fundamental differences between how researchers in psychology and mathematics education view and investigate the fundamental relations between spatial and mathematical reasoning. However, these differences provide fertile ground for exciting new investigations as each field respectively has advanced knowledge in some areas while leaving gaps in others. The commentaries are a starting point for identifying these points of contact and complementarity. We encourage readers to reflect on how the research in the two fields might be further integrated and how to build productive collaborations between the two sets of researchers. We thank all of our authors for taking a first step in this direction.

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Contents

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