International Handbook of Mathematical Learning Difficulties

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Editors

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From the Laboratory to the Classroom

Foreword by Brian Butterworth

Springer

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To our children
Lukas, Ramiro, Olavo, Catarina, Venla,
Anttoni, Aleksi and Viola

Foreword

Mathematical attainment is important for both individuals and societies. Despite widespread understanding that this is the case, systematic reviews of the impact of poor numeracy on life outcomes are relatively scarce and confined to the more advanced economies. In 2008, the UK Government Office for Science published a massive, authoritative and carefully researched report, the *Foresight Mental Capital and Wellbeing Project* subtitled *Making the Most of Ourselves in the 21st Century*. It was led by a very distinguished board of scientists and the government's Chief Scientific Officer, Sir John Beddington. The report summarised the consequences of very low numeracy, dyscalculia, which affects between 4% and 7% of children. Dyscalculia "has a much lower profile than dyslexia but can also have substantial impacts: it can reduce lifetime earnings by £114,000 and reduce the probability of achieving five or more GCSEs (A*-C) by 7–20 percentage points." [GCSEs are the main 16-year-old exam, and a requirement for further or higher education, and most decent jobs.] A large cohort study by the National Research and Development Centre in the UK concluded that men and women with poor numeracy have poorer educational prospects, earn less and are more likely to be unemployed, more likely to be in trouble with the law and more likely to be sick physically and mentally.

The consequences for society are also dramatic. Again for the UK, the accountancy firm, KPMG, estimated the cost to the UK of poor maths in terms of lost direct and indirect taxes, unemployment benefits, justice costs and additional educational costs was £2.4 *billion* per year. In 2011, the OECD's report, *The High Cost of Low Educational Performance*, demonstrated that the standard of maths drives GDP growth: the standard in 1960 was a good predictor of economic growth up to 2000; and the improvement in educational standard from 1975 to 2000 was highly correlated with improvement in economic growth. In particular, the report looked at the potential effects of improving standards in maths.

OECD's PISA (Programme for International Student Assessment) defines poor numeracy as "Level 2" and below, which means that at best children can only manage simple calculations with whole numbers. 11% of UK children fail to reach 400 PISA test points, the minimum level (which is not very high) for a numerate society. So, for example, the economic report found that if the UK improved the performance of those 11% from below minimum level to minimum level, the effect on the gross domestic product (GDP) would be a growth of about 0.44%. Not much you might think, but with an average rate of GDP growth of 1.5%, this would be a massive and cumulative increase of nearly one-third. The average long-run annual improvement in economic growth for all OECD countries (i.e. the richest countries) was 0.68%.

It is clear from international comparisons that there are enormous national differences in average levels of mathematical attainment and also in the proportion of children who are effectively innumerate in a way that affects their life chances and the health of their society more generally. According to the most recent study carried out by the OECD, "23% of students in OECD countries, and 32% of students in all participating countries and economies, did not reach the baseline Level 2 in the PISA mathematics assessment of 15 year olds. At that level, students can only extract relevant information from a single source and can use basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers" (OECD 2016, p4). This failure to reach Level 2 and be effectively innumerate varied from 3.8% in Shanghai China to 74.6% in Peru. In the USA, it is 25.8%, and in the UK it is 21.8%.

So what leads to the debilitating effects of low numeracy? There are, of course, many factors.

How children begin school is known to affect how they will continue. Canadian scientists put it like this, "Children who start school with poor knowledge and skills in … numeracy … are unlikely to catch up to their peers. Individual differences in … numeracy skills are evident at school entry—prior to formal instruction—suggesting that children acquire fundamental skills at home." All this is well-known and has particular relevance to mathematical development. Our study, carried out in Italy, revealed that the number and perhaps type of numerical activities in the home of the pre-schooler seems to be a key factor, and though this is related to parental income and education, especially maternal education, it constitutes a separate driver of early attainment. These factors have been systematically investigated in large-scale international comparisons like PISA.

Valuable and important as these international comparisons are, not to mention their political influence, there are two critical aspects of mathematical cognition and mathematical education that they are not designed to investigate, and perhaps cannot be designed to investigate:

- Explore the interaction of cognitive factors underlying mathematical attainment, especially in specific cultural and educational contexts
- Suggest particular methods of improving educational outcomes

Filling these important gaps makes this volume a vital and necessary complement to TIMSS and PISA.

Typical mathematical development depends on two distinct cognitive sources. One is domain-general capacities that affect almost all aspects of education. These will include reasoning and spatial abilities, long- and short-term memory, attention and motivation. Individual differences in these capacities can make a big difference to mathematical development over and above the cultural and educational context in which the learner is located. However, arithmetical competence is also based on domain-specific innate foundations that we share with other creatures. That is, without formal education, instruction of any kind, a wide range of creatures, from primates to birds to fish and even to insects, have been found to be able to extract information about the number of relevant objects in their environment. Lions can assess the number of invaders to their territory and decide whether there are enough of their own pride to fight them off, or whether they should retreat to fight another day. It is possible to train fish to swim to the larger (or smaller) of two arrays of abstract shapes. Bees count landmarks to estimate the distance between a food source and the hide.

Progress in mathematics and, especially, in numeracy depends on all these cognitive capacities.

Less well studied are the innate bases of geometry, but there is good evidence that human groups without access to geometrical education have good Euclidean intuitions.

This volume is unique in being able to locate the roles of these cognitive capacities, and incapacities, in their specific national contexts. It also offers an unparalleled perspective of how to assess mathematical development and to identify learners with atypical development in a way that respects cultural differences. Finally, it suggests ways in which the slow developers and the maths anxious can be helped. It is to be hoped that practitioners and policy-makers will read these essays carefully and be guided by the wealth of evidence herein provided.

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"Expect anything worthwhile to take a long time" (Debbie Millman).

This maxim also applies to our book. Writing and gathering the different contributions took much longer than expected, but the work on this book has proven to be an adventurous journey around the world together with multiple experts in the field of mathematical learning difficulties. The present editors are grateful for having experienced this and would firstly like to thank each of the numerous authors who have enhanced this book with their contributions. Different theoretical concepts, explanatory approaches and research findings allow for a profound insight into the topic of mathematical difficulties and largely encompass different perspectives of various countries.

Some more people have accompanied and supported us on our journey, helping to shape our project. At this point, we would like to thank them cordially.

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Alina Simon, University of Duisburg-Essen, took responsibility for the final editing of the book and prepared the manuscript for the official handover to the publisher. With remarkable competence and patience, she cared about all the aspects that often go unnoticed. We would like to sincerely thank her for this. Also, we would like to thank Moritz Herzog, University of Duisburg-Essen, who has volunteered for numerous responsibilities and coordinated a team of assistants in Essen.

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Annemarie Fritz is full professor for psychology at the University of Duisburg-Essen, Germany, since 1998. She graduated in psychology and habilitated in psychology of special education and rehabilitation. Since 2015, she is distinguished visiting professor at the University of Johannesburg.

At the University of Duisburg-Essen, she runs a research laboratory for children with learning difficulties. In the past 25 years, her research turned to children with mathematical learning difficulties. Here, the focus of her scientific work was the empirical validation of a development model of key numerical concepts and arithmetic skills from ages 4 to 8.

Based on this model, some diagnostic assessments (MARKO-Series) and training programmes for preschool and elementary school-children were developed. In cooperation with the University of Johannesburg, the MARKO-test for preschoolers and first graders was validated and normed in four South African languages. Recently, her interest turned to mathematical assessments for older children and mathematical anxiety.

Vitor Geraldi Haase is full professor of psychology at the Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil. He graduated in medicine, did his medical residency in paediatric neurology and has an M.A. in applied linguistics and a Ph.D. in medical psychology (Ludwig-Maximilians-Universität zu München). Working at UFMG since 1995, he heads the Laboratory for Developmental Neuropsychology and Número, a clinic for mathematical learning difficulties. He has been doing clinical work and research on numerical cognition applied to mathematical learning difficulties for the last 10 years. The main focus of this research is the characterisation of the molecular genetic variability underlying the cognitive mechanisms associated with mathematical learning difficulties and mathematical anxiety.

Pekka Räsänen is clinical neuropsychologist, principal investigator and executive vice director in the Niilo Mäki Institute, Jyväskylä, Finland. After his graduation, he has worked as a junior researcher in the Academy of Finland and as an interim associate professor of psychology in the University of Lapland, but since 1997 he worked as a researcher in the Niilo Mäki Institute. The Niilo Mäki Institute is a non-profit foundation-based research and development centre on learning and learning disabilities. The Niilo Mäki Institute is the largest and most influential research unit on learning disabilities in Finland offering also clinical services, further education and a publication unit. Pekka has developed most of the standardised tests of mathematical disabilities in Finland, and together with his colleagues, he has developed intervention programmes on language and mathematics for early education and computer-assisted tests and rehabilitation programmes for mathematical learning disabilities and recently also to visuospatial skills.