Quantitative Perspectives on Behavioral Economics and Finance

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The economic enterprise has firmly established itself as one of evaluating human responses to scarcity not as a rigidly rational game of optimization, but as a holistic behavioral phenomenon. The full spectrum of social sciences that inform economics, ranging from game theory to evolutionary psychology, has revealed the extent to which economic decisions and their consequences hinge on psychological, social, cognitive, and emotional factors beyond the reach of classical and neoclassical approaches to economics. Bounded rational decisions generate prices, returns, and resource allocation decisions that no purely rational approach to optimization would predict, let alone prescribe. Behavioral considerations hold the key to long-standing problems in economics and finance. Market imperfections such as bubbles and crashes, herd behavior, and the equity premium puzzle represent merely a few of the phenomena whose principal causes arise from the comprehensible mysteries of human perception and behavior. Within the heterodox, broad-ranging fields of behavioral economics, a distinct branch of behavioral finance has arisen. Finance has established itself as a distinct branch of economics by applying the full arsenal of mathematical learning on questions of risk management. Mathematical finance has become so specialized that its practitioners often divide themselves into distinct subfields. Whereas the *P* branch of mathematical finance seeks to model the future by managing portfolios through multivariate statistics, the *Q* world attempts to extrapolate the present and guide risk-neutral management through the use of partial differential equations to compute the proper price of derivatives. The emerging field of behavioral finance, worthy of designation by the Greek letter psi (ψ), has identified deep psychological limitations on the claims of the more traditional *P* and *Q* branches of mathematical finance. From Markowitz's original exercises in mean-variance optimization to the Black-Scholes pricing model, the foundations of mathematical finance rest on a seductively beautiful Gaussian edifice of symmetrical models and crisp quantitative modeling. When these models fail, the results are often catastrophic. The ψ branch of behavioral finance, along with other "postmodern" critiques of traditional financial wisdom, can guide theorists and practitioners alike toward a more complete understanding of the behavior of capital markets. It will no longer suffice to extrapolate prices and forecast market trends without validating these techniques according to the full range of economic theories and empirical data. Superior modeling and data-gathering have made it not only possible, but also imperative to harmonize mathematical finance with other branches of economics. Likewise, if behavioral finance wishes to fulfill its promise of transcending mere critique and providing a more comprehensive account of financial markets, behavioralists must engage the full mathematical apparatus known in all other branches of finance. In a world that simultaneously lauds Eugene Fama's efficiency hypotheses and heeds Robert Shiller's warnings against irrational exuberance, progress lies in Lars Peter Hansen's commitment to quantitative rigor. Theory and empiricism, one and indivisible, now and forever.

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Nicholas L. Georgakopoulos

Illustrating Finance Policy with *Mathematica*

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To mental health professionals,
whose value is underscored
by the apparent utter
rationality of finance.

Foreword

Seeing Is Believing

Truth routinely manifests itself through mathematics. "[T]he real world may be understood in terms of the real numbers, time and space and flesh and blood and dense primitive throbbings sustained somehow and brought to life by a network of secret mathematical nerves …."[1] Just as law uses words to animate the "enterprise of subjecting human conduct to the governance of rules,"[2] "[n]ature talks to us in the language of mathematics."[3] Through this volume, *Illustrating Finance Policy with* Mathematica, Nicholas L. Georgakopoulos enriches the series, *Quantitative Perspectives on Behavioral Economics and Finance*, by demonstrating the mathematical underpinnings of law and finance in vivid, visual terms.

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[1] David Berlinski, A Tour of the Calculus, at xiii (1996).

[2] Lon L. Fuller, The Morality of Law 122 (rev. ed. 1969).

[3] Peter Hilton, *The Mathematical Component of a Good Education*, *in* Miscellanea Mathematica 145–554, 149 (Peter Hilton, Friedrich Hirzebruch & Reinhold Remmert eds., 1991); *accord* Peter Hilton, *Foreword: Mathematics in Our Culture*, *in* Jan Gullberg, Mathematics: From the Birth of Numbers, at xvii–xxii, xix (1997).

Professor Georgakopoulos seeks to provide "a visual explanation" of concepts such as "the CAPM [capital asset pricing model] or the call options pricing formula" in intuitive and memorable form. Indeed, he acknowledges that this book, at its "most basic," enables the reader "to learn the fundamental concepts of modern finance without the quantitative foundation that most finance books require," by "getting the intuitions visually from the graphics." To do so, however, neglects this book's full quantitative and computational potential. This preface therefore encourages readers to aspire, however slowly, toward Prof. Georgakopoulos's "more demanding approach" of learning "how to use Mathematica in simple finance applications and in the production of graphics."

Both Prof. Geogakopoulous and I hold academic appointments as professors of law. This volume treats lawyers, law students, and their instructors as members of its target audience. Admittedly, "[i]t is an open secret that lawyers" (stereo)typically "don't like math."[4] In a legal culture whose leaders shamelessly confess their ignorance of the "fine details of molecular biology,"[5] lawyers and lawmakers run a dire risk of falling behind "the extraordinary rate of scientific and other technological advances that figure increasingly in litigation" and, for that matter, in daily life.[6]

Even though law addresses subjects "so vast that fully to comprehend [them] would require an almost universal knowledge ranging from geology, biology, chemistry and medicine to the niceties of the legislative, judicial and administrative processes of government,"[7] the "extraordinary condition" of the legal profession "makes it possible for [someone] without any knowledge of even the rudiments of chemistry to pass

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[4] Lisa Milot, *Illuminating Innumeracy*, 63 Case W. L. Rev. 769–812, 769 (2013).

[5] Association for Molecular Pathology v. Myriad Genetics, Inc., 133 S. Ct. 2107, 2120 (2013) (Scalia, J., concurring in part and concurring in the judgment) ("I join the judgment of the Court, and all of its opinion except Part I-A and some portions of the rest of the opinion going into fine details of molecular biology. I am unable to affirm those details on my own knowledge or even my own belief").

[6] Jackson v. Pollion, 753 F.3d 786, 788 (7th Cir. 2013) (Posner, J.).

[7] Queensboro Farms Prods., Inc. v. Wickard, 137 F.2d 969, 975 (2nd Cir. 1943) (describing agriculture, particularly dairy farming).

upon" scientifically or technologically sophisticated questions.[8] No less than other social sciences, law should aspire to a level of "numeracy," one that is "less about numbers per se and more about statistical inference or how to interpret and understand scientific … studies."[9]

A mastery of basic mathematical concepts should serve as a foundation for serious legal scholarship. As a group that not only digests but also delivers postmodern criticism,[10] legal scholars can surely grasp mathematics, which after all is merely another branch of philosophy.[11] "Legal reasoning," in particular, represents merely a special case of "*theory construction*."[12] The prospect of teaching machines to speak a language that expresses and conveys legal knowledge fulfills the aesthetic if not the practical goals of information theory.[13]

Despite the law's reputation for quantitative ineptitude, mathematical thinking naturally suits this discipline. Social scientists have nurtured "something like a third culture" between science and literature in order to improve the circumstances under which real "human beings are living."[14] Accordingly, scholars within this tradition enjoy a special

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[8] Parke-Davis & Co. v. H.K. Mulford Co., 189 F. 95, 115 (S.D.N.Y. 1911) (Hand, J.).

[9] Edward K. Cheng, *Fighting Legal Innumeracy*, 17 Green Bag 2d 271–78, 272 (2014); *accord* United States *ex rel.* Customs Fraud Investigations, LLC. v. Victaulic Co., 839 F.3d 242, 270 & n.56 (3rd Cir. 2016), cert. denied, 138 S. Ct. 107 (2017). *See generally* John Allen Paulos, Innumeracy: Mathematical Illiteracy and Its Consequences (2nd ed. 2001) (1st ed. 1988).

[10] *Cf.* Stanley Eugene Fish, Doing What Comes Naturally: Change, Rhetoric, and the Practice of Theory in Literary and Legal Studies (1990).

[11] *See* U.S. Patent & Trademark Office, General Requirements Bulletin for Admission to the Examination for Registration to Practice in Cases Before the United States Patent and Trademark Office 37 (2008) (describing mathematics as a philosophical discipline and therefore insufficient by itself to satisfy the technical training requirement for eligibility to take the Patent and Trademark Office examination); *see also* 37 C.F.R. § 11.7(a)(2)(ii) (requiring practitioners before the USPTO to "[p]ossess the legal, scientific, and technical qualifications necessary … to render … valuable service" to patent and trademark applicants).

[12] L. Thorne McCarty, *Some Arguments About Legal Arguments*, *in* Sixth International Conference on Artificial Intelligence and Law 215–24, 221 (1997) (emphasis in original).

[13] *See generally* Abraham Moles, Information Theory and Esthetic Perception (Joel E. Cohen trans., 1968); Frieder Nake, Ästhetik als Informationsverarbeitung (1974) ("Aesthetics as Information Processing").

[14] C.P. Snow, The Two Cultures: And a Second Look 70 (2nd ed. 1965).

opportunity to unite the literary culture's "canon of works and expressive techniques" with the scientific culture's "guiding principles of quantitative thought and strict logic."[15] At their best, social scientists bridge all of contemporary civilization's intellectual subcultures.[16]

The application of empirical methods to discrete problems is perhaps the most familiar and deeply rooted form of mathematically informed social science. More than a century after Oliver Wendell Holmes declared that "the man of the future is the man of statistics and the master of economics,"[17] and three decades after Richard Posner celebrated the decline of law as an autonomous discipline,[18] empiricism dominates contemporary legal studies.

Law and finance face staggering amounts of data and fierce competition among analytical models. Imperfectly articulated hypotheses in these branches of social science will likely lead neither to elegant closed-form solutions nor to pathological functions. Even the most thoughtfully elaborated empirical tests fail to deliver complete knowledge about law and its underlying logic. "Every year, if not every day, we have to wager our salvation upon some prophecy based upon imperfect knowledge."[19]

Mathematical analysis in social science typically follows a dialectic of romance, frustration, and eventual reconciliation of internal logic with

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[15] Frank Wilczek, *The Third Culture*, 424 Nature 997–98, 997 (2003).

[16] *Cf. Cultural Divides, Forty Years On*, 398 Nature 91, 91 (1999) (recognizing how C.P. Snow's depiction of two cultures "still resonates" in a world "where cultural antipathies are very much alive and kicking").

[17] Oliver Wendell Holmes, *The Path of the Law*, 10 Harv. L. Rev. 457–78, 470 (1897), *reprinted in* 110 Harv. L. Rev. 991–1009, 1001 (1997).

[18] *See generally* Richard A. Posner, *The Decline of Law as an Autonomous Discipline: 1962–1987*, 100 Harv. L. Rev. 761–80 (1987).

[19] Abrams v. United States, 250 U.S. 616, 630 (1919) (Holmes, J., dissenting).

external reality. All of science follows a familiar progression. "Normal science does not aim at novelties of fact or theory and, when successful, finds none."[20] But "fundamental novelties of fact and theory" trigger "the recognition that nature has somehow violated the paradigm-induced expectations that govern normal science."[21] Once an "awareness of anomaly ha[s] lasted so long and penetrated so deep" as to plunge a scientific discipline into "a state of growing crisis," a succeeding "period of pronounced professional insecurity" over "the persistent failure of the puzzles of normal science" prompts a fruitful search for new rules.[22] Wonderfully, "random shocks"—the impetus for progress in economics, law, and science—happen to be "the subject matter" of finance and its regulation.[23]

Mathematics serves as a source of beauty and sensory delight. Uniquely among human endeavors, mathematics boasts "a beauty cold and austere, … without any appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry."[24] As the poet Edna St. Vincent Millay expressed the sentiment: "Euclid alone has looked on Beauty bare."[25] What Justice Potter Stewart said of obscenity (that he knew it when he saw it)[26] finds a parallel in Paul Erdős's definition of

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[20] *See* Thomas S. Kuhn, The Structure of Scientific Revolutions 52 (2d ed. enlarged, 1970).

[21] *Id.* at 52–53.

[22] *See id.* at 66–67.

[23] John Y. Campbell, *Asset Pricing at the Millennium*, 55 J. Fin. 1515–67, 1515 (2000).

[24] Bertrand Russell, *The Study of Mathematics*, *in* Mysticism and Logic, and Other Essays 58–73, 60 (1988); *accord* Jim Chen, *Truth and Beauty: A Legal Translation*, 41 U. Toledo L. Rev. 261–67, 265 (2010).

[25] Edna St. Vincent Millay, *Euclid Alone Has Looked on Beauty Bare*, *in* Selected Poems 52 (J.D. McClatchy ed., 2003).

[26] *See* Jacobellis v. Ohio, 378 U.S. 184, 197 (1964) (Stewart, J., concurring) ("[P]erhaps I could never succeed in intelligibly [defining obscenity]. But I know it when I see it ….").

mathematical beauty: "Why are numbers beautiful? It's like asking why Beethoven's Ninth Symphony is beautiful. If you don't see why, someone can't tell you."[27]

Seeing is believing. So is hearing. Standard mathematical specifications of human sensory perception, particularly sight and sound, literally round out our world. For example, the HSV and HSL models project color onto cylindrical or conic space where *hue* (distinctive shades spanning red, magenta, blue, cyan, green, and yellow) is represented as a circular spectrum; *saturation* (the richness of color, or its chroma) is represented as linear or angular distance from a neutral, gray axis; and *value* indicates the degree of pure brightness or its opposite, darkness, relative to the origin.[28]

Music can be represented in a similar physical space. If intensity of a tone is indicated along a line segment of length 1 from the threshold of hearing (0 dB) to the threshold of pain (130 dB),[29] a corresponding angle of $\pi/2$ radians within a "unit cone" can indicate frequency across the range of human hearing, 20–20,000 Hz.[30] Twelve equally spaced notes within each pitch class (a single octave, separated by the one above it by a doubling of frequency and the one below it by a halving of frequency) appear at an interval of 100 cents within the appropriately named *chromatic scale*.[31] As sight or sound, color and music are mathematics made flesh.[32]

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[27] Paul Hoffman, The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth 42 (1998) (quoting Erdős); *accord* Keith Devlin, The Math Gene: How Mathematical Thinking Evolved and Why Numbers Are Like Gossip 140 (2000).

[28] *See* Alvy Ray Smith, Color gamut transform pairs, 12(3) Computer Graphics 12–19 (August 1978) (describing the "hexcone" model of HSV colorspace); George H. Joblove & Donald Greenberg, *Color Spaces for Computer Graphics*, 12(3) Computer Graphics 20–25 (August 1978) (describing the HSL model and comparing it to HSV); *cf.* Dorothy Nickerson, *History of the Munsell Color System*, 1 Color Research & Applic. 121–30 (1976). *See generally* Steven K. Shevell, The Science of Color 202–06 (2d ed. 2003).

[29] *See*, e.g., David Howard & Jamie Angus, Acoustics and Psychoacoustics § 2.3, at 80–82 (2012); Debi Prasad Tripathy, Noise Pollution 35 (2008).

[30] *See generally* Harry F. Olson, Music, Physics and Engineering 248–51 (1967).

[31] *See* 1 Bruce Benward & Marilyn Saker, Music: In Theoryand Practice 47 (7th ed. 2003).

[32] *Cf.* John 1:14 ("And the Word was made flesh, and dwelt among us….").

The most beautiful mathematical results exhibit "a very high degree of unexpectedness, combined with inevitability and economy."[33] Deep beauty subsists in connections that first appear unrelated,[34] but upon further examination reveal "metaphoric combination[s]" that "leap[] beyond systematic placement" and "explore[] connections that before were unsuspected."[35] The most "useful and fertile combinations" of ideas "present themselves to the mind in a sort of sudden illumination, after an unconscious working somewhat prolonged," and ultimately "seem the result of a first impression."[36]

The real world, however, often inconveniently fails to align itself with mathematically beautiful models. In the face of anomalous results, even the most rigorous, comprehensively elaborated approach "can no longer understand [itself] because the theories … of [a] former age no longer work and the theories of the new age are not yet known."[37] That challenge leaves exactly one path forward: to "start afresh as if [we] were newly come into a new world."[38]

Financial economics has undergone a crisis of precisely this sort. Much of contemporary mathematical finance, from the Capital Asset Pricing Model (CAPM) to the Black–Scholes model of option pricing,[39] Merton's distance-to-default model of credit risk,[40] the original

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33 G.H. Hardy, A Mathematician's Apology 29 (1940).

34 *See* Gian-Carlo Rota, The Phenomenology of Mathematical Beauty 173 (1997).

35 Jerome S. Bruner, *The Conditions of Creativity*, *in* On Knowing: Essays for the Left Hand 17–30, 20 (1963).

36 Henri Poincaré, *Mathematical Creation*, *in* The Foundations of Science: Science and Hypothesis, The Value of Science, Science and Method 383–94, 391 (George Bruce Halstead trans., 1913).

37 Walker Percy, *The Delta Factor*, *in* The Message in the Bottle: How Queer Man Is, How Queer Language Is, and What One Has to Do with the Other 3–45, 3 (1986).

38 *Id.* at 7.

39 *See* Fischer Black & Myron S. Scholes, *The Pricing of Options and Corporate Liabilities,* 81 J. Pol. Econ. 637–54 (1973); Robert C. Merton, *The Theory of Rational Option Pricing*, 4 Bell J. Econ. 141–83 (1973).

40 *See* Robert C. Merton, *On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 29 J. Fin. 449 (1974).

RiskMetrics specification of value-at-risk,[41] and the Gaussian copula,[42] is built on the Gaussian "normal" distribution.[43]

These elegant models—absent elaborate modifications that ruin their spare, symmetrical form—are treacherously wrong in their reporting of the true nature of risk. Many of the predictive flaws in contemporary finance arise from reliance on the mathematically elegant but practically unrealistic construction of "beautifully Platonic models on a Gaussian base."[44] Gaussian mathematics suggests that financial returns are smooth, symmetrical, and predictable. In reality, returns are skewed[45] and exhibit heavier than normal tails.[46]

The attraction in law and finance to formal elegance reflects a love affair with the Gaussian mathematics that has traditionally dominated

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[41] *See* Jorge Mina & Jerry Yi Xiao. Return to RiskMetrics: The Evolution of a Standard (2001); Jeremy Berkowitz & James O'Brien, *How Accurate Are Value-at-Risk Models at Commercial Banks?,* 57 J. Fin. 1093–111 (2002).

[42] *See* Roger B. Nelsen, An Introduction to Copulas (1999); David X. Liu, *On Default Correlation: A Copula Function Approach*, 9(4) J. Fixed Income 43–54 (March 2000).

[43] *See generally* Benoit B. Mandelbrot & Richard L. Hudson, The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward (2004).

[44] Nassim Nicholas Taleb, The Black Swan: The Impact of the Highly Improbable 279 (2007).

[45] *See, e.g.*, John Y. Campbell, Andrew W. Lo & A. Craig MacKinlay, The Econometrics of Financial Markets 17, 81, 172, 498 (1997); Felipe M. Aparicio & Javier Estrada, *Empirical Distributions of Stock Returns: European Securities Markets, 1990–95*, 7 Eur. J. Fin. 1–21 (2001); Geert Bekaert, Claude Erb, Campbell R. Harvey & Tadas Viskanta, *Distributional Characteristics of Emerging Market Returns and Asset Allocation*, 24(2) J. Portfolio Mgmt. 102–116 (Winter 1998); Pornchai Chunhachinda, Krishnan Dandepani, Shahid Hamid & Arun J. Prakash, *Portfolio Selection and Skewness: Evidence from International Stock Markets*, 21 J. Banking & Fin. 143–67 (1997); Amado Peiró, *Skewness in Financial Returns*, 23 J. Banking & Fin. 847–62 (1999).

[46] *See*, e.g., J. Brian Gray & Dan W. French, *Empirical Comparisons of Distributional Models for Stock Index Returns*, 17 J. Bus. Fin. & Accounting 451–59 (1990); Stanley J. Kon, *Models of Stock Returns—A Comparison*, 39 J. Fin. 147–65 (1984); Harry M. Markowitz & Nilufer Usmen, *The Likelihood of Various Stock Market Return Distributions, Part 1: Principles of Inference*, 13 J. Risk & Uncertainty 207–19 (1996); Harry M. Markowitz & Nilufer Usmen, *The Likelihood of Various Stock Market Return Distributions, Part 2: Empirical Results*, 13 J. Risk & Uncertainty 221–47 (1996); Terence C. Mills, *Modelling Skewness and Kurtosis in the London Stock Exchange FT-SE Index Return Distributions*, 44 Statistician 323–34 (1995). *See generally* Terence C. Mills, The Econometric Modelling of Financial Time Series (2nd ed. 1999).

the culture of the natural and social sciences.[47] Grasping the uncomfortable truth that Gaussian models of risk and return belong to "a system of childish illusions" forces our infatuation with the seductive symmetry of traditional risk modeling to pass "like first love … into memory."[48]

This conflict is often portrayed as an irreconcilable struggle between the romance of beauty and the realism of truth. Hermann Weyl admonished physics (and presumably all other pursuits informed by mathematics) that any necessary choice between truth and beauty should favor beauty.[49] Practical versus philosophical "conflict" over "the purpose of scientific inquiry" is "an ancient [struggle] in science."[50] Although law ordinary seeks "knowledge … for purely practical reasons, to predict and control some part of nature for society's benefit," the knowledge unveiled through mathematical analysis "may serve more abstract ends for the contemplative soul" and "[u]ncover[] new relationships" that prove "aesthetically satisfying" insofar as they "bring[] order to a chaotic world."[51]

This tension is illusory. Mathematics itself delivers an elegant denouement. Mathematical analysis is ordinarily associated with—indeed, often equated with—the application of established empirical techniques to ever-growing bodies of data. This book, however, demonstrates that the quantitative visualization of social phenomena enjoys a far broader scope and entertains vastly deeper ambitions. In stark "contrast with soulless calculation," "[g]enuine mathematics … constitutes one of the finest expressions of the human spirit."[52]

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[47] *See* Nassim Nicholas Taleb, The Black Swan: The Impact of the Highly Improbable 279 (2007).

[48] Berlinski, *supra* note 1, at 239.

[49] Freeman J. Dyson, *Prof. Hermann Weyl*, 177 Nature 457–58, 458 (1956) (quoting Weyl: "My work always tried to unite the true with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."); *accord* Edward O. Wilson, Biophilia 61 (1984).

[50] Sharon E. Kingsland, Modeling Nature: Episodes in the History of Population Ecology 4–5 (1985).

[51] *Id.* at 4–5.

[52] Hilton, *The Mathematical Component of a Good Education*, *supra* note 3, at 151; *accord* Hilton, *Mathematics in Our Culture*, *supra* note 3, at xxi.

The "great areas of mathematics"—including "combinatorics, probability theory, statistics," and other fields of greatest interest to social science—"have undoubtedly arisen from our experience of the world around us."[53] Law and finance apply mathematical tools "in order to systematize that experience, to give it order and coherence, and thereby to enable us to predict and perhaps control future events."[54] But scientific progress responds to "what might be called the mathematician's apprehension of the natural dynamic of mathematics itself."[55]

With unmatched power and intuitive appeal, visual demonstrations such as those provided throughout *Illustrating Finance Policy with* Mathematica show precisely how "law is not indifferent to considerations of degree."[56] The very responsiveness of quantitative measures to changing conditions—an admittedly "qualitative notion" called "robustness"[57]—is "essential for law enforcement."[58] Robustness ensures that "different judges will reach similar conclusions" when they encounter similar data.[59]

Mathematics *as doing* delivers the answers that we most pressingly seek—not simply according to the data describing the world as we find it, but also according to the own internal logic of mathematics. "[T]here is nothing in the world of mathematics that corresponds to an audience in a concert hall, where the passive listen to the active. Happily, mathematicians are all *doers*, not spectators."[60] Through its

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[53] *Id.*

[54] *Id.*

[55] *Id.*

[56] Schechter Poultry Corp. v. United States, 295 U.S. 495, 554 (1935) (Cardozo, J., concurring).

[57] Rama Cont, Romain De Guest & Giacomo Scandolo, *Robustness and Sensitivity of Risk Measurement Procedures*, 10 Quant. Fin. 593–606, 594 (2010).

[58] Steven Kou, Xianhua Peng & Chris C. Hyde, *External Risk Measures and Basel Accords*, 38 Math. Operational Research 393–417, 401 (2013).

[59] *Id.*

[60] George M. Phillips, Mathematics Is Not a Spectator Sport, at vii (2005).

quest for "universal interest[s]," Prof. Georgakopoulos's introduction to finance and its visual representation may yet "catch an echo of the infinite, a glimpse of its unfathomable process, a hint of the universal law."[61]

James Ming Chen

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Special thanks to Heather Elaine Worland Chen.

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[61] Holmes, *supra* note 17, at 478, *reprinted in* 110 Harv. L. Rev. at 1009.

Preface

Purpose: Visual Help

The purpose of this book is not to explain these concepts from beginning to end. Rather, I think of this book as a companion to advanced law, finance, and policy courses where students that meet new concepts need a little more of a visual explanation of those concepts. Courses discuss the Capital Asset Pricing Model (CAPM) or the call options pricing formula as mathematical concepts and equations. Most students miss the intuitive appeal of those concepts. I think that seeing the CAPM evidence in a graph (Fig. 6 in Chapter 5, p. 67) or the wedge-like solid that approximates the call option valuation formula (Fig. 4 in Chapter 6, p. 82) makes a big difference. Yet those figures appear here for the first time—I have seen them in none of the books that try to introduce these concepts.

Researchers who already understand the intuitions behind the graphs can use the book's discussion of the production of graphics. This book explains the financial usage of graphical illustrations, their intuitive appeal, and their production. The Mathematica file that produces the figures of this book is available on my personal website (nicholasgeorgakopoulos.org) under scholarship, at the entry that corresponds to this book.

The Quantitative Finance Core

The core of this book consists of the three chapters on quantitative finance. Discounting is the object of Chapter 4. The Capital Asset Pricing Model is the object of Chapter 5. The call option valuation formula is the object of Chapter 6. Granted, these are complex matters. In a classroom setting, these topics can take far more time than the corresponding number of pages may indicate.

Whereas the mathematics are complex, they are not voluminous nor do they require mathematical manipulations. My experience is that students who enjoy math, even if they do not recall nor have had any math after high school, can handle the related concepts and formulas because the intuitions come visually from the graphs.

Instructors can elect how to use the rest of the book depending on their goals. Statistically-oriented courses would tend to opt for the chapter on illustrating statistics (Chapter 7) and probability theory (Chapter 8). Corporate courses would tend to want to include the chapter on financial statements (Chapter 9). Courses focusing on making decisions would want to include the chapter on aversion toward risk (Chapter 10). Courses that wish to focus on economic concepts would tend to include the chapter on financial crises (Chapter 11). The foundational concept of the need to justify law on a failure of the market from the perspective of the analysis of Coase (Chapter 1) is too abbreviated here for students who truly encounter it for the first time and likely would need additional supportive material (such as the two-chapter treatment it receives in my Principles and Methods of Law and Economics, 2005) if an instructor truly intends to focus on it.

The Chapters on the Mathematica Software and Math Concepts Are Not Necessary

One can read this book with several goals. The most basic one is to learn the fundamental concepts of modern finance without the quantitative foundation that most finance books require, by getting the intuitions visually from the graphics. A more demanding approach is to learn also how to use Mathematica in simple finance applications and in the production of graphics.

In the former case, the chapters explaining Mathematica and mathematical concepts (Chapters 2 and 3) are not necessary, they can be skipped, and when, in the other chapters, the exercises call for using Mathematica, readers can try using the program of their choice. When the exercise does not call for solving an equation in symbolic form, a spreadsheet program would often suffice. The online simplified version of Mathematica at wolframalpha.com can act as a supplement, providing the capacity to solve equations in symbolic form. Other programs with the capacity for symbolic algebra, such as Maple, would also be adequate.

Legal Citation Style

This book uses the legal style of citations. This primarily means that the volume number precedes the name of the publication, that the year comes after the full citation, and that a parenthetical explaining the relevance of the cited matter follows the citation. For example, whereas in the social sciences citation style a journal article would appear as author (year) title of article, publication name volume:page, this would appear here as author, *title of article*, volume Publication Name page (year) (explanatory parenthetical). Notice that articles are italicized and books in small capitals.

Acknowledgements

First, I wish to thank my research assistants in this project, Allan Griffey, Henry Robison, and John Millikan. No less important has been the help that I have received from my students who questioned and struggled with these concepts over the many years of my teaching financial concepts in my various courses. For Mathematica programming help, I must acknowledge the enormous help from numerous users of the

Mathematica group of StackExchange, mathematica.stackexchange.com. For useful comments I wish to thank George Triantaphyllou, Stephen Utz and the Oral Argument Podcast for Christian Turner's and Joe Miller's enthusiastic comments about a part of this project.

Special thanks for bringing this book to light are due to Dean and Prof. Jim Chen, the editor of this series, and Elizabeth Graber, Palgrave Macmillan's Acquisitions Editor, Assistant Editor Allison Neuburger, and Azarudeen Ahamed Sheriff, the book's production manager. Prof. Chen deserves an additional round of thanks for his colorful foreword.

Vail, Colorado, USA Nicholas L. Georgakopoulos

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