Michell Structures

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In memory of
Professor George Rozvany

Preface

The publication written by Anthony George Maldon Michell (1904) was far ahead of its time. This remarkable paper has posed and solved selected optimum design problems in which a structure as a whole is treated as the design variable, contrary to the traditional setting in which only some selected dimensions or parameters may be chosen to play this role. Michell has raised the question of a safe and economic way of transmitting the given loading to the prescribed zone of the support, without imposing *a priori* assumptions on the layout of the bars (i.e. on the structural topology concerning the position of nodes and members connections) designed to make this load transmission possible. The adopted safety conditions require keeping the uniaxial stress in members between the lower bound $-\sigma_C$ and the upper bound σ_T. The cost of the structure is assumed as proportional to its volume.

Forty years later, Leonid Kantorovich (1942) has shed a new light on the mass transport problem of Gaspard Monge. Only in 2001, Guy Bouchitté and Giuseppe Buttazzo noted essential links between the Monge–Kantorovich—and the Michell problems. Although these two problems do not coincide, both share the same theoretical background. To pose the problems correctly, one should construct the same kind of abstract variational settings expressed in terms of signed Radon measures.

Thus, a correct setting of the Michell problem necessitates an abstract reformulation. Michell's paper starts from referring to James C. Maxwell's results on truss design. Michell adopted Maxwell's concept of the embedding of a structure in a field $\mathbf{v}$ representing virtual displacements. The values of this field at the nodes are interpreted as their virtual displacements. Since the field $\mathbf{v}$ fills up the whole design domain, frequently being the whole space, the concept of introducing such a field is set in an abstract manner; the virtual displacements at nodes are in no connection with nodal displacements caused by the given load. Michell discovered that for the lightest, fully stressed structure, one can indicate a virtual displacement field $\mathbf{v}$ in which this structure is embedded such that: (i) this field makes the value of the virtual work of the load maximal, and (ii) this field generates virtual relative elongations of truss members lying within the bounds: $\frac{-\sigma_0}{\sigma_C}$, $\frac{\sigma_0}{\sigma_T}$; the referential stress σ_0 may be assumed arbitrarily.

It is really surprising that the problem of the volume minimization leads to the problem of maximization of the virtual work. Moreover, the problem set originally for trusses is rearranged to a problem of optimum design of continuum bodies. The continuum setting is the result of the phenomenon of the decrease in the volume along with increasing the number of possible truss members. This process has no its natural end, thus leading to the optimum structures being continuum bodies of fibrous microstructure. Indeed, if the number of possible members increases, the truss members can be formed according to a better layout contributing to a smaller value of the volume. Since no lower limit on the areas of the member cross sections is imposed, the members may be thinner and thinner thus allowing the total volume being smaller and smaller. The optimal Michell structures assume the form of discrete-continuum structures, thus extending the initial class of truss structures.

The next theoretical step is dualization of the mentioned problem of maximization of the virtual work leading to the problem of minimization of a certain measure of the stress field equilibrating the given loading and transmitting this loading to the prescribed support zone of the continuum body considered; minimization is taken over all statically admissible stress fields. This measure of stress fields was not known to Michell; it was proposed much later by George Rozvany (1976) in the context of flexural problems, and then rediscovered (for the case of the permissible stresses in tension and compression being equal) by Gilbert Strang and Robert Kohn (1983). A complete set of mathematical theorems, including both the maximization and minimization problems, can be found in the seminal paper by Guy Bouchitté, Wilfrid Gangbo and Pierre Seppecher (2008), where, probably for the first time, not only Michell's theory but even the linear theory of statics of trusses have been formulated there from scratch in terms of a fully mathematical language. Thus, the Michell problem reduces to the two, mutually dual problems whose solutions are linked by the optimality conditions. The main property of the optimum design is a pointwise locking of the virtual strains which naturally links this theory with William Prager's (1957) theory of deformations of bodies made of materials with locking.

The theory of Michell's structures would be only a forgotten artefact if the theory were not augmented and illustrated by carefully chosen examples, representing the exact and clear illustrative solutions to sufficiently rich class of fundamental problems of structural analysis. Spectacular is admitting the point loads, impossible to handle properly in elasticity. Indeed, the elasticity theory does not encompass the material concentrations on surfaces and curves, thus not allowing for the presence of the point loads as well the loads applied along the curves. The available Michell's structures, i.e. the exact solutions to Michell's optimum design problems, have been an inspiration for structural and aircraft engineers, architects and machine designers for the last decade and found applications even in the modern areas like mechatronics and biotechnology. One of the most important problems is to find the Michell structure carrying three self-equilibrated forces in the plane. The main family of the exact solutions to this problem, for the case of equal permissible stresses in tension and compression, has been constructed by Henry Chan (1966), but this report has been concealed for 40 years, being not mentioned even in the fundamental book by Walter Hemp (1973). In 2006, Prof. Henry Chan has generously sent this report to one of the authors of the present book, which has paved the way towards constructing new exact solutions composed of the substructures being the solutions to the three-force problems.

The exact solutions to Michell's problems are both educating and inspiring. In particular, they show that the edges should be strengthened by ribs; but they disclose also the necessity of using the ribs inside the structure. The optimal structures show also very fine methods of surrounding the point loads by fibrous substructures to minimize displacements of the nodes where the point loads are applied. The exact solutions constitute certain families of layouts; an exact solution may change its topology from one layout to the other, depending on the data. Small changes of the data may cause drastic differences in the optimum layouts. This phenomenon resembles the theory of attractors in the non-linear dynamics. Due to these subtle properties of the exact solutions, a precise numerical method for predicting new solutions or verifying the old solutions is pending. Let it be stressed here that all the exact solutions to Michell's problems reported in the present book have been checked numerically by the highly precise version of the ground structure method. This method has been proposed by W. S. Dorn, Ralf E. Gomory and Harvey J. Greenberg (1964), Matthew Gilbert and Andy Tyas (2003) and then improved by Tomasz Sokół (2011).

An anti-plane counterpart of the Michell problem in the plane is the optimum design of grillages of rigid joints and systems of beams, as set up by William Prager and George Rozvany, see Rozvany (1976) for the rich bibliography. Although the theory of optimal layout of grillages looks similar to Michell's theory, this similarity is only apparent. The properties of the exact solutions are different; in particular, the fibres in the optimum grillages are never curved. The mathematical theory of grillage optimization is being elaborated by Karol Bołbotowski; the results presented in this book are published by his courtesy.

The theories of Michell's structures and Prager–Rozvany grillages are prototypes of the contemporary field of optimization, called optimization of structural topology, or, simply topology optimization. This field, well represented in the books by Martin Bendsøe (1995) and Martin Bendsøe and Ole Sigmund (2003), develops rapidly, mostly by the activity of the Society of Structural and Multidisciplinary Optimization (ISSMO), the main organizer of the series of the biennial World Congresses on Structural and Multidisciplinary Optimization (WCSMO) and the patron of the journal: Structural and Multidisciplinary Optimization, edited by Springer-Verlag since 1989.

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