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Finite Elements for Truss and Frame Structures

An Introduction Based on the Computer Algebra System Maxima

Springer

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Only a generation of readers will spawn a generation of writers.

Steven Spielberg

Preface

This book is intended as a study aid for the finite element method. Based on the free computer algebra system Maxima, we offer routines to symbolically or numerically solve problems from the context of plane truss and frame structures. This allows to check classical 'hand calculations' on the one hand and to understand the computer implementation of the method on the other hand. The mechanical theories focus on the classical one-dimensional structural elements, i.e., bars, Euler–Bernoulli and Timoshenko beams as well as their combination of generalized beam elements. Focusing on one-dimensional elements reduces the complexity of the mathematical framework, and the resulting matrix equations are still possible to be displayed with all components and not only in a symbolic representation. The use of a computer algebra system and the incorporated functions, e.g., for equation solving, allows to focus more on the methodology of the finite element method and not on standard procedures. Some of the provided examples should be also solved in a classical 'hand calculations' to better understand the computer implementation.

We look forward to receiving some comments and suggestions for the next edition of this textbook.

Esslingen am Neckar, Germany — Andreas Öchsner
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Contents

Symbols and Abbreviations

Latin Symbols (Capital Letters)

A	Area, cross-sectional area
E	YOUNG's modulus
EA	Tensile stiffness
EI	Bending stiffness
F	Force
G	Shear modulus
GA	Shear stiffness
I	Second moment of area
$\boldsymbol{K}$	Global stiffness matrix
$\boldsymbol{K}^{\mathrm{e}}$	Elemental stiffness matrix
L	Element length
M	Moment
N	Normal force (internal), interpolation function
$\boldsymbol{N}$	Column matrix of interpolation
Q	Shear force (internal)
$\boldsymbol{T}$	Transformation matrix
X	Global Cartesian coordinate
Y	Global Cartesian coordinate
Z	Global Cartesian coordinate

Latin Symbols (Small Letters)

a	Geometric dimension
$\boldsymbol{f}$	Global column matrix of nodal loads
$\boldsymbol{f}^{\mathrm{e}}$	Elemental column matrix of nodal loads
k_{s}	Shear correction factor
m	Element number
n	Node number
p	Distributed load in x-direction

q Distributed load in y-direction
u Displacement
$\boldsymbol{u}$ Global column matrix of nodal deformations
$\boldsymbol{u}^{\mathrm{e}}$ Elemental column matrix of nodal deformations
x Cartesian coordinate
y Cartesian coordinate
z Cartesian coordinate

Greek Symbols (Small Letters)

α Rotation angle, factor
β Angle
γ Shear strain (engineering definition),
ε Strain
κ Curvature
ν Poisson's ratio
σ Normal stress
τ Shear stress
ϕ Rotation (Timoshenko beam)
φ Rotation (Bernoulli beam)

Mathematical Symbols

$\times$ Multiplication sign (used where essential)
A Assembly operator

Indices, Superscripted

$\ldots^{\mathrm{e}}$ Element
$\ldots^{\mathrm{R}}$ Reaction

Indices, Subscripted

$\ldots_{\mathrm{p}}$ Point

Abbreviations

1D One-dimensional
CAD Computer-aided design
ECA Element connectivity array
EFA Element freedom array
FEM Finite element method