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Nathan Sidoli • Yoichi Isahaya

Thābit ibn Qurra's Restoration of Euclid's *Data*

Text, Translation, Commentary

Springer

Nathan Sidoli
School for International Liberal Studies
Waseda University
Tokyo, Japan

Yoichi Isahaya
College of Arts
Rikkyo University
Tokyo, Japan

ISSN 2196-8810 ISSN 2196-8829 (electronic)
Sources and Studies in the History of Mathematics and Physical Sciences
ISBN 978-3-319-94660-3 ISBN 978-3-319-94661-0 (eBook)
https://doi.org/10.1007/978-3-319-94661-0

Library of Congress Control Number: 2018951065

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

"… for the towering dead
With their nightingales and their psalms"

Preface

This book began in a reading group lead by myself and attended by Takatomo Inoue, Yoichi Isahaya, and Masayo Watanabe. After reading a few other things, we began to read Thābit's version of Eucid's *Data* from images of two manuscripts. After a month or two had passed, Inoue and Watanabe moved on to bigger and better things, and Isahaya and I decided to see the text through. As we progressed, I began to think, naively, that we could fairly easily turn this work into an edition of the text—maybe with a translation. Our working practice was that Isahaya would prepare a draft of the text, send it to me, I would prepare a draft of the translation, then we would meet and correct both against the manuscripts. We also compared this material—at first somewhat sporadically—with al-Ṭūsī's version of the treatise. As the project developed, I slowly came to see that in order to do it justice, we would need to produce a monograph study, including a new interpretation of the Greek text.

A number of scholars have helped us in various ways as we worked on this project. Fabio Acerbi, Mohammad Bagheri, Hamid Bohlul, Benno van Dalen, Richard Lorch, and Ken Saito all kindly aided us in securing copies of manuscripts. Andrew Arana and Marco Panza discussed philosophical matters with Nathan Sidoli, both in person and in correspondence, and thus helped us to clarify our thinking on a number of points. Alexander Jones made a number of corrections and suggestions for clarification in the editorial process. Fabio Acerbi made many critical comments on the introduction and saved us from a number of blunders. Sonja Brentjes and Takanori Suzuki both read the complete manuscript and made many comments and corrections. Tim Hicks proofread the English prose. We are extremely grateful for their generosity.

This book was typeset using XƎTEX. We are grateful to the many developers of this project, and especially to Lars Madsen and Peter Wilson for their work on the memoir package, and to Vafa Khalighi for his work on the bidi package. The font for the Roman script is Peter Baker's Junicode; that for the Arabic script is Khaled Hosny's Amiri; and that for the Greek script is Gentium Plus. These fonts are available under the Open Font License. The dedication is a quotation from "In My Craft or Sullen Art" by Dylan Thomas.

Nathan Sidoli
Tokyo, November 20, 2017

Mathematical notation

In order to discuss the mathematical content of the work, we introduce the following notational conventions:[1]

Magnitudes: We denote magnitudes with a blackboard-bold type, such that $\mathbb{A}$ denotes a general magnitude and $\mathbb{a}$ denotes the same magnitude when it is known—that is, known in magnitude.

Points: We denote points with italic type, such that A denotes a general point, and a denotes the same point when it is known—that is, known in position. Hence, we can also denote a general line as AB, and the same line as AB_p, AB_m or $AB_{p,m}$ when it is known, since a line can be known in either position, magnitude, or both. In this way, aB is the name of any line passing through the given point a_p, while aB_p is the name of a certain line given in position that passes through the given point a_p—the label B, however, is just part of the name of the line and does not indicate any particular point. The name $ab_{p,m}$ denotes a line whose endpoints are both given in position.[2]

Lines: We also use cursive type to denote lines with a single letter, such that $\mathcal{L}$ denotes a general line, and ℓ_p, ℓ_m or $\ell_{p,m}$ denotes the same line, as known in position, in magnitude, or in both.

Figures: We denote rectilinear figures with bold type, such that a general, rectilinear figure, constructed from points A, B, C, ... is denoted as $\mathbf{F}(ABC...)$, or from lines $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ as $\mathbf{F}(\mathcal{ABC}...)$, a triangle as $\mathbf{T}(ABC)$, a square as $\mathbf{S}(ABCD)$, $\mathbf{S}(AB)$, or $\mathbf{S}(\mathcal{A})$ a rectangle as $\mathbf{R}(AB, BC)$, or $\mathbf{R}(\mathcal{A}, \mathcal{B})$, a parallelogram as $\mathbf{P}(ABCD)$, $\mathbf{P}(AB, BC)$, or $\mathbf{P}(\mathcal{A}, \mathcal{B})$, and a gnomon as $\mathbf{G}(ABCDEF)$ or $\mathbf{G}(AB, BC, CD, AF)$. A rectilinear figure can be known in magnitude, $\mathbf{F}(ABC...)_m$, in form, $\mathbf{F}(ABC...)_f$, or both, $\mathbf{F}(ABC...)_{f,m}$.

Circles: A circle is denoted as $\mathbf{C}(A, \mathcal{R})$, where A is its center and $\mathcal{R}$ its radius; $\mathbf{C}(\mathcal{R})$, where $\mathcal{R}$ is its radius, or as $\mathbf{C}(ABC)$, where A, B and C are three points through which it passes. A circle may be known in magnitude, $\mathbf{C}(A, r_m)_m$, or in magnitude and in position, $\mathbf{C}(a, r_m)_{m,p}$.

Angles: We use Greek letters to denote angles, such that Θ denotes a general angle, and θ_m, $\theta_{m,p}$, denote an angle known in magnitude, or known in magnitude and in position. We also often denote an angle with three points, $\angle ABC$ and $\angle ABC_m$.

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[1] Our notation is modeled on that introduced by Dijksterhuis (1987, 51–52) in the 1938 Dutch original of his work on Archimedes, and uses some aspects of a notation employed by Taisbak (1991).

[2] The distinction between $AB_{p,m}$ and $ab_{p,m}$ may seem pedantic, but the *Data* also deals with lines given in position and magnitude whose endpoints are not themselves given, such as that cut off between two parallel lines given in position on a line that falls on them at a given angle—consider the implication of *Data* 27.

Ratios: A ratio between two magnitudes $\mathbb{A}$ and $\mathbb{B}$ is denoted $(\mathbb{A} : \mathbb{B})$, and when it is known it is denoted $(\mathbb{A} : \mathbb{B})_r$.

These symbols are simply a shorthand for the concepts introduced verbally in the text. In particular, they are designed to avoid the ambiguities that can arise from using the algebraic symbols introduced by Descartes that distinguish clearly between known and unknown quantities and lend themselves to an arithmetic reading.

For example, using modern algebraic notation, one might express Prop. 2—if there is a known magnitude and its ratio to another magnitude is known, then the other magnitude is known—as something like

$$a, a : x = r \Rightarrow x = ra.$$ [3]

In this case, however, it is difficult not to read this symbolic expression as arithmetic, despite the fact that the proposition is meant to cover a broader range of interpretations.

In our symbolism, Prop. 2 would be expressed as

$$\mathbb{a}, (\mathbb{a} : \mathbb{B})_r \Rightarrow \mathbb{b},$$

which is nothing more than a shorthand for the verbal expression given above. In particular, we are interested in expressing the fact that the two *b*s denote the same magnitude, first in general and then as known, and that the second *b* is known despite the fact that it is not expressed in terms of known magnitudes given in the enunciation of the proposition, which is simply what we read in the ancient proposition.

[3] For example, Thaer (1962, 66) summarizes this proposition as "Von a, κ aus κa"—which the above stated expression fleshes out.

Naming conventions

We refer to the individual definitions and propositions of Thābit's *Restoration of the Data* as Def. x_a and Prop. x_a, while those of the Greek *Data*, as edited by Menge (1896), are referred to as *Data* Def.x_g, *Data* x_g. For example, Prop. 52 (*Data* 54), refers to the 52nd proposition of the *Restoration*, as edited and translated in this work, corresponding to the 54th proposition of Menge's edition. We will often refer to the Arabic work studied in this book simply as the *Restoration*.

Transcription of letter-names

For translations of Greek and Arabic mathematical sources, we use the following transcriptions for letter-names. All translations are our own.

Arabic		**Greek**	
ا	A	α	A
ب	B	β	B
جـ	G	γ	G
د	D	δ	D
ه	E	ε	E
ز	Z	ζ	Z
ح	H	η	H
ط	T	θ	Q
ک	K		
ل	L	λ	L
م	M		
ن	N		
س	S		
ص	C		

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