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From Classical to Modern Analysis

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To Blanquita

Preface

Why Another Analysis Book? There are many good undergraduate and graduate analysis books. There is, however, a rather large gap between undergraduate and graduate texts. The purpose of this book is to bridge this gap.

Typically, a graduate book covers measure theory and the Lebesgue integral. An undergraduate book does not. A book covering measure theory usually assumes that the reader is familiar with limits superior and inferior, arithmetic operations involving infinity, set theory, metric spaces, and some topology. Many students get into a measure theory course with only a vague notion of these concepts. Even more importantly, the level of abstraction the students have been exposed to is much lower than what is required to succeed in a graduate analysis course. As a result measure theory courses are much harder for the student than they need to be. The main goal of this book is to propose a gently rising path from undergraduate analysis (i.e., Classical Analysis) to measure theory (i.e., Modern Analysis).

Prerequisites We assume the reader of this text to have had an advanced calculus course at the level of Fitzpatrick (2006) or Schinazi (2011). In particular, the reader should be able to write and read short mathematical proofs. We also assume familiarity with the main results of the calculus of one variable. However, we do review most of what is needed as we go along and this book is largely self-contained.

What Courses Is This Book For? We have used these lecture notes for two semester long analysis courses for advanced undergraduate and beginning graduate students.

In the first semester we cover the first seven chapters. These chapters cover standard material such as number systems, convergence of sequences and series as well as more advanced topics such as limits superior and inferior, convergence of functions, and metric spaces.

In the second semester we cover Chapters 8 through 12 and selected topics from the last chapters. These last chapters are largely independent. The main purpose of the second course is an introduction to measure theory and the Lebesgue integral as well as a few applications.

Analysis is the theory of calculus. To be complete this theory requires measure theory and the Lebesgue integral. Most books at this level do not cover measure theory but choose to revisit calculus of several variables instead. In our experience, more calculus at this point does not do much for the student. Introducing measure theory on the other hand opens a whole new world.

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