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Truth, Existence and Explanation

FilMat 2016 Studies in the Philosophy of Mathematics

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Preface

The philosophy of mathematics aims at understanding the wonderful phenomenon of mathematics, and its immense resonance, from a philosophical point of view. This multifaceted intellectual enterprise is large enough to require a wide vista of present-day mathematics and its practice and to be constrained by matters of epistemology, metaphysics, logic, history of mathematics, and philosophy of language, mind, and science. However, it is also precisely because of its distinctive capacity of crossing disciplinary boundaries that the philosophy of mathematics has reached a sophisticated level of autonomy under the mantle of philosophy, with its own goals and standards of success. Nevertheless, autonomy does not mean insularity. Indeed, the outcome of the application of formal methods and techniques to a repertoire of vexed ontological and epistemological questions about mathematics encroaches on philosophy *sans phrase* as well as it deserves mathematical audience. This is what the present volume essentially aspires to.

The chapters here collected, in particular, zoom in on assorted themes concerning the triumvirate *truth*, *existence*, and *explanation* in mathematics, trying to move forward current debates in the literature along different axes of point of view. Needless to say, the intimate relation between truth, existence, and explanation propels another batch of interrelated issues for the philosopher: truth serves explanatory purposes, and successful mathematical explanations of the physical reality have been traditionally considered as a path toward arguing for the existence of mathematical objects.

Most of these contributions originate from the Second FilMat Conference that took place at the University of Chieti-Pescara from May 26 to 28, 2016. This conference hosted 19 talks and received 36 submissions from 32 international universities and research institutions from Austria, Belgium, Brazil, Canada, France, Germany, Hungary, India, Israel, Italy, Mexico, Poland, USA, and UK. FilMat conferences are organized under the aegis of the Italian Network for the Philosophy of Mathematics, FilMat Network (www.filmatnetwork.com). The network now counts almost 70 Italian scholars worldwide in the philosophy of mathematics and closely related disciplines. The present book follows the volume *Objectivity, Realism, and Proof* edited by F. Boccuni and A. Sereni in 2016 in the same Springer series, which selected contributions from the First FilMat Conference held at San Raffaele University, Milan, in May 29–31, 2014.

There is no doubt that the "space" of a collective volume is continuous, not discrete. So, in subdividing this kind of book into distinct parts there always remains a residue of arbitrariness on the editor's part. The chapters of this volume are self-contained, for the most part. The chapters in Part I deal with a group of issues concerned with mathematical truth, touching on problems such as the question of how to conceptualize the truth value of undecidable sentences, the intricacies of the interaction between absolute provability and truth theories, the motivation for a deflationary account according to which there is no substantial property of truth, and the important but elusive role of intensionality in mathematics.

This is a synopsis of the individual chapters.

Enrico Moriconi's contribution "Some Remarks on *True* Undecidable Sentences" considers the classical problem of assigning a truth value to Gödel's independent sentences. In particular, Moriconi's proposal is to examine the trajectory of this question in relation to three different levels: ordinary mathematics, informal theories, and formal theories such as Peano Arithmetic. The author argues that these three levels should not be treated separately inasmuch as it is only when one considers their constant interplay that a proper solution to the problem of the truth value of Gödel's sentences emerges.

Johannes Stern, "Penrose's New Argument and Paradox," examines the contemporary assessments of Penrose's New Argument for the claim that the human mind cannot be mechanized. In particular, Stern shows that Penrose's argument cannot be formalized within a theory of truth and absolute provability, in the sense that there is no consistent theory that allows for a formalization of the argument in a straightforward way. In a second step, the author sets up a reasonable theory of truth and absolute provability in which Penrose's argumentative strategy leads to a sound argument but, as a more general conclusion, he shows that the argument relies on a pathological feature of the theory.

Carlo Nicolai in "On Expressive Power Over Arithmetic" examines the relationship between a syntactical base theory and extensions by modal predicates and/or truth predicates. For the syntactical base theory, the author takes into account sequential theories which fall into two different kinds, either inductive or finitely axiomatizable. As for the relationship between the modal theory and its base theory, Nicolai considers three notions that have been suggested in the literature: conservativeness, relative interpretability, and speed up. He shows that many of the results known are highly dependent on the choice of the base theory. The author therefore questions some of the philosophical conclusions that have been too hastily drawn arguing that a more fine-grained analysis is needed.

Jaroslav Peregrin, in his chapter "Intensionality in Mathematics," addresses a key question in the philosophy of mathematics: Can we make room for *intensions* in mathematics? That question is specifically treated in the light of the fact that only *extensions* seem to be meaningfully associated with mathematical expressions, according to the standard possible-worlds based account of intensions. However, the author claims that we cannot give up intensionality in the metamathematical talk and so the possible-worlds account needs to be suitably modified. Peregrin's final proposal is that intensions should be taken as corresponding to rules regulating how (much) extensions are allowed to change without infringing the "boundaries of identity."

Denis Bonnay and Henri Galinon in "Deflationary Truth Is a Logical Notion" examine the thesis according to which "truth" should be taken as a *logical* notion. Such a thesis was inaugurated by Quine and now endorsed by some contemporary deflationists such as Horwich and Field. In their paper, the authors supply new arguments in favor of the logical nature of the notion of "truth" by focusing on Tarski's invariance approach to logicality.

Andrea Strollo's chapter "Making Sense of Deflationism from a Formal Perspective: Conservativity and Relative Interpretability" purports to import and interpret technical results in the study of formal theories of truth into the classical philosophical debate in which the notion of "truth" is taken metaphysically. In particular, the author discusses the role of the formal notions of *conservativity* and *relative interpretability* in the evaluation of the deflationary nature of "truth". The final result of Strollo's approach is a new taxonomy of axiomatic theories of truth.

The chapters in Part II are more heterogeneous. They explore a variety of issues concerning the ubiquitous notion of structure in mathematics, the nature of mathematical understanding, the explanatory role of completeness theorem and inductive proofs, the interplay between the informal side of computability and the formal one, and the effectiveness and applicability of mathematics. The last chapter of the book is meant to be a measure of the sophistication of ancient topics in the philosophy of mathematics, such as Aristotelian view of continuity.

Reinhard Kahle, in "Structure and Structures," takes the view that mathematical truth makes sense only insofar as it refers to structures. However, the author observes that it is far from being obvious how these structures, together with their internal structure, are given. By taking a different point of view with respect to the standard debate on mathematical structuralism, he addresses the question of how the informal talk about "structures" in ordinary mathematics is included in the notion of "structure" as it is technically defined in mathematical logic. To this purpose, the author distinguishes between *first-order*, *primitive*, and *abstract* structures.

Janet Folina in "Towards a Better Understanding of Mathematical Understanding" examines the general notion of *understanding* in mathematics. This chapter incorporates two main thesis. According to the first, mathematical understanding should be considered as a "family of resemblance" type of concept, and this explains why it is in general very hard to frame it into a well-defined theory. Secondly, the notion of "mathematical structure" can help in filling the conceptual gap between the different epistemological meanings associated with "understanding."

In the chapter "The Explanatory Power of a New Proof: Henkin's Completeness Proof," **John Baldwin** embarks on a detailed comparison of the Henkin and Gödel proofs of the completeness theorem. The author's main purpose here is to underline the explanatory power of Henkin's argument as both a proof of the completeness of first-order logic and, more in generality, show how this argument allowed for the entire recasting of model-theory as an autonomous and flourishing mathematical discipline.

In her chapter “Can Proofs by Mathematical Induction Be Explanatory?,” **Josephine Salverda** argues against the received view according to which proofs by induction usually lack explanatory value. In particular, Marc Lange recently maintained that inductive proofs can never be explanatory. Against this latter strong claim, Salverda’s chapter aims to provide examples of explanatory inductive proof, together with a positive suggestion for making sense of these examples on Lange’s account.

In the chapter “Ontological Commitment and the Import of Mathematics,” **Daniele Molinini** critically focuses on a recent article by Alan Baker in which the author argues that there is a class of explanations in science, namely optimization explanations, for which the use of more general mathematical resources leads to a reduction of concrete commitments and to a boost in the explanatory power. Molinini challenges this thesis in the context of Baker’s preferred example involving periodical cicadas, thus raising doubts against the alleged import of Baker’s considerations in the Platonism vs. Nominalism debate.

In his chapter “Applicability Problems Generalized,” **Michele Ginammi** aims to lay the foundation for a general philosophical account of *applicability*. To this end, the author provides interesting case studies concerning all the possible relevant applicability configurations involving mathematics and physics. Finally, Ginammi tries to “extract” a general pattern common to each one of the math-to-physics, math-to-math, and physics-to-math applications. The kind of generality he finally achieves allows for a better understanding of the complex interaction between physics and mathematics.

Luca San Mauro’s chapter “Church-Turing Thesis, in Practice” provides the reader with a philosophical account of the notion of “proof by Church’s Thesis”; this is the notion which allows mathematicians to resort to informal methods when working in computability theory. Firstly, the author offers a detailed reconstruction of the historical development of this notion, from Post to Rogers. Then, in contrast with the received view on the topic, San Mauro shows how the *informal* side of computability is not completely reducible to its *formal* counterpart.

The last chapter in this volume is devoted to Aristotle’s conception of the relationship between existence and conceivability. In “Existence vs. Conceivability in Aristotle: Are Straight Lines Infinitely Extendible?,” **Monica Ugaglia** argues that Aristotle’s philosophy is consistent with the mathematics of its time. In particular, the chapter shows how Aristotle, resorting to the infinite procedure of division of the continuum, accounted for the infinity of numbers and asymptotic properties, while at the same time maintaining that mathematical entities are immanent in physical ones and that the physical world is finite.

Pisa, Italy — Mario Piazza
Caparica, Portugal — Gabriele Pulcini
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