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Editors

# Superschool on Derived Categories and D-branes

## Edmonton, Canada, July 17–23, 2016

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# Preface

String Theory revolutionized not just how we view the physical world but also how we view Mathematics. Conversely, through String Theory, many physicists first became acquainted with beautiful fields of Mathematics, like Algebraic Geometry. The cross-pollination of insights and motivations between String Theory and Mathematics led to remarkable insights in both fields.

One such deep instance is that of Mirror Symmetry, a duality in String Theory that provides a powerful computational tool—allowing one to exchange difficult computations for simpler ones. The full range of consequences of Mirror Symmetry in Mathematics may never be understood. On the other hand, Mirror Symmetry has already provided spectacular insight in enumerative geometry [1] leading to a revolution in the field [2–6]. Two related mathematical proposals for Mirror Symmetry arose afterward. The Strominger–Yau–Zaslow or SYZ conjecture [7] posits that mirror manifolds arise from the process of T-dualization; each space admits torus fibrations over a common base, and the exchange between the two amounts to dualization of the torus fibers. The Homological Mirror Symmetry of Kontsevich [8] states that an equivalence of categories underlies all phenomena of Mirror Symmetry. It provides a deep and hitherto-unknown connection between the fields of Algebraic Geometry and Symplectic Geometry and has become a robust field of Mathematics itself in a short time.

This book consists of a series of introductory lectures on Mirror Symmetry and its surrounding topics. These lectures were provided by participants in the PIMS Superschool School for Derived Categories and D-Branes in July 2016. Together, they form a comprehensive introduction to the field which integrates perspectives from mathematicians and physicists alike.

The intent is to provide a pleasant and broad introduction into modern research topics surrounding String Theory and Mirror Symmetry which is approachable to readers who are new to the subject. Mathematical readers should expect to come away with a broader perspective on this field and a bit of physical intuition. Physicists will gain an introductory overview of the developing mathematical realization of physical predictions. Topics include constructions of various mirror pairs, approaches to Mirror Symmetry, connections to homological algebra, and physical motivations.

Of particular interest is the connection between GLSMs, D-branes, birational geometry, and derived categories. This is one of the broader themes of the text and is explained from a physical and mathematical perspective. The introductory lectures provided herein highlight many features of this emerging field and give concrete connections between the physics and the math.

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# Contents

# Symbols for “Abelian and Triangulated Categories”

Chantelle Hanratty[1]

1. $\mathscr{C}$ or $\mathscr{D}$: A specific category
2. Ob($\mathscr{C}$): The objects in the category $\mathscr{C}$
3. $\mathrm{Hom}_{\mathscr{C}}(A, B)$ or $\mathrm{Hom}(A, B)$: Morphisms (in the category $\mathscr{C}$) between the objects $A$ and $B$
4. $\cong$: Isomorphic
5. $F^{-1}$: The inverse functor to a functor $F$
6. $A[n]$: The object $A$ shifted $n$ times in a triangulated category; $T^n(A)$
7. $f[n]$: The map $T^n(f)$: $A[n] \to B[n]$, where $f : A \to B$.
8. $f_*, f^*$: If $f$: $A \to B$, then $f_*$ and $f^*$ are the induced maps between morphism groups $\mathrm{Hom}(X, A) \to \mathrm{Hom}(X, B)$ and $\mathrm{Hom}(B, X) \to \mathrm{Hom}(A, X)$ respectively.

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