

Advanced Structured Materials

Volume 91

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Indentation Testing of Biological Materials

 Springer

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To our families for their continuous support

Preface

Indentation technique is a very simple method to try mechanical properties of both natural and artificial materials. It can be performed with the help of a rigid body, called indenter, and is aimed at quantifying a material's resistance, for instance, to external loading. The main difficulty in using indentation testing is to find the way how to interpret the measured indenter displacement in terms of the material properties. To answer this question, one needs not only a mathematical model for characterizing the material deformation but also to solve the respective contact problem of interaction between the indenter and the surface of a material sample. The latter problem is the simplest one, when the contact area is relatively small compared with the sample's size, so that sample size effects can be neglected. In the case of biological materials, other issues, such as anisotropy, friction, adhesion, or time-dependent response, should be taken into account, and they are the main focus of the present study. We recommended this monograph for researchers and engineers dealing with different aspects of indentation testing as well as for Ph.D. students enrolled in contact mechanics and biomechanics courses.

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