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# A Mathematical Primer on Quantum Mechanics

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*To Ade*

# Preface

This book is an elaboration of lecture notes written during the preparation of courses for mathematics students given for some years at the Universities of L'Aquila and "La Sapienza" of Roma. The aim of the book is to provide an elementary introduction to nonrelativistic Quantum Mechanics for a single particle from the point of view of Mathematical Physics. It is mainly addressed to students in Mathematics at master's degree level, but it could also be useful for students in Physics interested in a deeper understanding of the mathematical aspects of the theory.

No previous physical knowledge of Quantum Mechanics is required and the mathematical language necessary to develop theory and applications is explicitly introduced. The prerequisites are basic notions of Classical Physics, elementary theory of Hilbert spaces, Fourier transform, and elements of real and complex analysis.

The first four chapters of the book are aimed to provide some physical and mathematical background. In Chap. 1, the basic notions of Hamiltonian Mechanics and Electromagnetism are recalled, with an emphasis on some aspects relevant for understanding the origin and the development of Quantum Mechanics. In Chap. 2, the historical evolution of the ideas in Physics from Planck's hypothesis to Bohr's model of the hydrogen atom is briefly described. In Chap. 3, we introduce the Schrödinger equation, discuss Born's statistical interpretation, and outline a first sketch of the new theory. It is also stressed the need of more advanced notions from the theory of linear operators in Hilbert spaces to obtain a rigorous and consistent description of the theory. Such notions are introduced in Chap. 4, where we describe the main concepts, i.e., self-adjoint and unitary operators, resolvent and spectrum, spectral theorem (stated without proof), different classifications of the spectrum, all supported by examples and exercises.

In Chap. 5, an axiomatic and rigorous formulation of nonrelativistic Quantum Mechanics is given. We avoid generality and neglect some technical difficulties. The aim is to provide the theoretical instruments to approach elementary problems of Quantum Mechanics for a single particle. We also briefly mention some interpretational problems of the theory connected with the measurement process.

The applications of the formalism are described in Chaps. 6–9. The emphasis is on a detailed analysis of simple models as free particle, harmonic oscillator, point interaction, and hydrogen atom. For each model, we construct the self-adjoint Hamiltonian, characterize the spectrum, and discuss the main dynamical properties. These models are of crucial importance to understand the qualitative behavior of a quantum particle in some relevant physical situations. We believe that a deep understanding of their physical and mathematical properties is an unavoidable prerequisite to deal with more difficult problems in Quantum Mechanics.

Some more advanced topics are also treated starting from the concrete analysis of the models. In Chap. 9, elements of spectral analysis for Schrödinger operators are introduced to study the spectrum of the Hamiltonian of the hydrogen atom. In Appendix A, the semiclassical evolution of a Gaussian state is described using the results obtained for the harmonic oscillator in Chap. 7. In Appendix B, the basic concepts of scattering theory are discussed and then used to revisit the simple one-dimensional scattering problem analyzed in Chap. 8 for a point interaction.

Many exercises are proposed in the text and their solution is crucial for a good understanding of the subject.

After studying this book, the reader should have gained the essential background to approach the analysis of advanced mathematical problems in Quantum Mechanics. Among the many excellent textbooks on the subject appeared in the literature in recent times, we mention

Strocchi, F.: An Introduction to the Mathematical Structure of Quantum Mechanics. World Scientific (2008).
Faddeev, L. D., Yakubovskii, O. A.: Lectures on Quantum Mechanics for Mathematics Students. AMS (2009).
Teschl, G.: Mathematical Methods in Quantum Mechanics. AMS (2009).
Dimock, J.: Quantum Mechanics and Quantum Field Theory. Cambridge University Press (2011).
Gustafson, S. J., Sigal, I. M.: Mathematical Concepts of Quantum Mechanics. Springer (2011).
Hall, B. C.: Quantum Theory for Mathematicians. Springer (2013).
Dell'Antonio, G.: Lectures on the Mathematics of Quantum Mechanics. Atlantis Press (2015).

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